THE ARITHMETIC and METRIC SYSTEM

made simple
FOR PRIMARY CLASSES
with comparisons
BETWEEN OLD ITALIAN PRICES AND MEASURES
AND THE SAME IN METRIC DECIMAL

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To the good reader

This small work has been reprinted many times and has become very widespread but there were no longer stocks left of earlier editions and it had not been further reprinted. At the invitation of many and distinguished people has it now been rewritten and published for rural schools, trade students and for general use in Primary Education following the Government's Education Department programmes.

Since many have done their schooling prior to the new system coming into force and others need to know the different systems because of business or employment, tables showing each one can allow someone to quickly see and compare the old systems of weights and measures in Italy with the new. Someone who needs to carry out these operations correctly will also find the conversion tables for many weights and measures and corresponding prices.

My aim was to be brief, clear and to help the children of common folk. If I have succeeded, then let it be to the glory of the One who gives us everything that is good; if not, then I beg the reader to accept my good intentions and bear with my efforts.

May you all live happily.

I. - Preliminary ideas and the number system.[1]

Q. What is arithmetic?

A. Arithmetic is the science of numbers.

Since numbers can be combined and broken down, arithmetic could be called the science of putting numbers together or breaking thenm into different parts.

Q. What is meant by number?

A. Number means union of units or parts of units.

Q. What is meant by quantity?

A. Quantity is whatever can be said to be greater or lesser:

The length of a road, the size of an army are quantity because they are of greater or lesser length or extension.

Q. What is a unit?

A. A unit is something on its own or considered on its own, e.g. *a book, an inkwell, a year, a table, a triangle, a kilogram, a people.*

Q. How are numbers formed?

A. Numbers are formed by putting units together, or by dividing a unit into parts.

So by adding one unit to another we get the number *two*; by adding another to that we get the number *three*; by dividing the unit into two we arrive at a *half*; by dividing it into three we have *thirds, fourths* etc. Thus we can end up with an endless series of numbers.

Q. How many kinds of numbers are there?

A. There are three kinds of numbers:

1. **Whole** numbers that contain a complete unit, so: *one, four, ten* etc.
2. **Fractions** which contain complete units and parts of a unit, e.g. *one and a half apples.*

[1] What is found in Chapters 1 and 2 is enough to satisfy 1st and 2nd year Primary programmes

3. **Fractions** that express only parts of a unit without any whole number. e.g.: *three quarters of an hour, half a pound*, etc.

Q. How can we classify numbers still further?

A. Numbers can be further classified into **abstract** and **concrete**. Abstract numbers are those which do not indicate the name of the species they belong to, e.g.: *twenty, forty, hundred.* Concrete numbers are those where we indicate the species which the number belongs to, such as *twenty years, three hours, a hundred students*, etc.

Q. How can we memorise the names of all whole numbers?

A. It would be very difficult to learn arithmetic if all numbers had a particular name. To make them easier to learn, numbers are combined in such a way that they can all be named with just a few terms. Numbers are divided into UNITS, TENS, HUNDREDS in such a way that ten units form a set of ten, ten sets of ten make a hundred and tend hundreds make a unit of a higher order that is called a THOUSAND. Ten of these units of a thousand make ten thousand, ten tens of thousands make a hundred thousand, ten hundred thousands makes a unit of a higher order than a thousand, that is, it becomes a MILLION. And so we continue on to BILLIONS, TRILLIONS etc. It is sufficient to know the names for the simple units, tens and hundreds to be able to say any number at all.[2]

Q. What are the names of the **units**, the **tens**, and the **hundreds**?

A. Units are as follows: ONE, TWO, THREE, FOUR, FIVE, SIX, SEVEN, EIGHT, NINE. The names of the tens are: TEN, TWENTY, THIRTY, FORTY, FIFTY, SIXTY, SEVENTY, EIGHTY, NINETY. The names of the hundreds are: ONE HUNDRED, TWO HUNDRED, THREE HUNDRED, FOUR HUNDRED, FIVE HUNDRED, SIX HUNDRED, SEVEN HUNDRED, EIGHT HUNDRED, NINE HUNDRED. Other than these names. there are proper names for the numbers between ten and twenty and these are: ELEVEN, TWELVE, THIRTEEN, FOURTEEN, FIFTEEN, SIXTEEN, SEVENTEEN, EIGHTEEN, NINETEEn. From what has been said up until now we can see that numbers are divided into various orders; UNITS, THOUSANDS, MILLIONS, BILLIONS etc. one being higher than the other, and that each order is subdivided into hundreds, tens and units.

Q. Is there any rule for saying these numbers?

A. We always begin with the largest order and gradually arrive at the smallest. In each order, then, we first of all say the hundreds, then the tens, then the units and follow that with the name of the order they belong to, leaving out the units, tens

[2] This property of numbering, by which ten units make tens, ten tens make hundreds etc, having the number ten as a base, means that this system of counting is called the *decimal system*, while other systems have other names. So a system which needs twelve units before moving to a higher order would be called a *duodecimal* system

and hundreds that are missing in any order. So if a number includes four tens and three hundred units and also six tens and five units of thousands, it is said in the following way; *sixty five thousand, three hundred and forty.*[3]

Q. How are numbers written?

A. Numbers are written with signs called **digits**.

Q. What are digits?

A. There are nine digits used for expressing numbers:

ONE	TWO	THREE	FOUR	FIVE	SIX	SEVEN	EIGHT	NINE
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

and these are called **significant** digits.

A. To these we add **O** (zero), a digit which in itself is insignificant, that is, it does not express any number but takes the place of digits that are missing.[4]

Q. How do we write numbers that go beyond nine units?

A. When writing any number we keep the same rule as we do when saying them, that is, we begin by writing the hundreds, the tens, the units of the higher order, then the hundreds, tens and units of the order immediately below, until we arrive at the simple units; be careful however to write zeros for the hundreds, tens or units missing in any of the orders.

So to write the number *thirty five thousand two hundred and six* in digits: begin by writing the higher order, which is the thousands. Since there are not hundreds of thousands, the three tens and five units of thousands are written, then immediately we write the order of units beginning by noting the two hundreds, a zero for the tens that are not there in the number given, so the six simple units; so we end up with 35, 206.[5]

Q. What is the rule for reading numbers?

A. The digits that make them up are separated into groups of three beginning from the right and moving left. The first group counts units, the second thousands, the third millions etc. The final group can have less than three digits. Then we begin from the left when we read the number contained in each group, as if it was on its own, adding at the end of each group the name of the order it represents.

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[3] When reading or writing whole numbers only there is no need to name the lower order which is understood easily enough from the higher order

[4] These digits are called Arabic because it is believed they were invented by the Arabs. They are also called Indian because some believe the Arabs got them from the Indians

[5] We do not put a zero at the beginning of the hundreds of thousands because in fact a zero in front of a whole number is useless, while a number would get smaller if we were to omit the zero in the middle or at the end of it.

So to read the number 31405078: dividing the number from right to left into groups of three digits we get:

millions	*thousands*	*units*
31	405	078
tens units	*hundreds tens units*	*hundreds tens units*

The first group on the right expresses the units, the second the thousands, the third the millions. Then beginning from the left, to read the number of each group as if it was on its own we say: thirty one million four hundred and five thousand and seventy eight.

Q. What values can a digit have?

A. Two: **absolute** and **relative**. **Absolute** value is the value a digit has in itself independent of where it is found. The **relative** value is the one the digit acquires according to the place it occupies when the number is written.

So for example: in the number 68 the absolute value of the first digit on the left is *six*, while the relative value is six tens or *sixty*.

Q. What do we call that part of arithmetic that teaches us how to form, read and write numbers?

A. It is called **numeration** or counting.

Q. So what is numeration and how is it divided?

A. Numeration is that part of arithmetic that teaches us how to form numbers and express them in words and represent them with written signs. So numeration can be **written** and **spoken**.

Q. What is **spoken numeration**?

A. **Spoken numeration** is the way we form numbers and express them with words.

Q. What is **written numeration**?

A. **Written numeration** is said to be the way we represent numbers with a handful of signs called digits.

Q. What are the basic operations in arithmetic?

A. The basic operations that form the basis of all arithmetic are: **addition, subtraction, multiplication** and **division**.

EXERCISES IN NUMERATION OR COUNTING.

Write:

Seventeen francs in digits.

A hundred and twenty five good young men.

One thousand two hundred shingles or tiles.

The city of Turin has around two hundred and twenty thousand inhabitants.

One thousand five hundred and six *myria* [not used in English but a *myria* is equivalent to ten kilograms] of wood;

Eighty thousand brave soldiers.

Read the following numbers:

800 *myriagrams* of grapes [note: *myriagram* = 10,000 grams].

At the battle of Lepanto Christians vanquished an army of more than 50000 Turks.

More than 18000000 Christians have given their lives for the Faith in persecutions.

II. - Addition.

Q. What is **addition**?

A. **Addition** is an operation by which we join two or more numbers of the same species to see how many they make when taken together. The numbers that are to be added are called *items*. The number that results from the union of items is called the *sum or total*.

Q. Can we add together numbers of different species?

A. Numbers of different species cannot be added together.

So, for example, if I say: 25 francs and 60 kilograms, we need to consider the sums separately. But if I say: 25 francs and 50 francs they can be added together because they are the same species.

Q. What do we need to be aware of when we add?

A. To do addition we need to be careful that the digits of the various items are written in such a way that the units are written under units, tens under tens, hundreds under hundreds, etc.

EXAMPLE: -
To write 513 and 85 we arrange the numbers thus:

First item	513
Second item	85

But we have to make sure the number 5 is written under 3 and 8 under the 1.

With the numbers arranged that way and a horizontal line underneath, the operation is done as follows:

First item	513
Second item	85
Horizontal line & Total	598

Begin from the right, that is from the units column saying simply: 5 plus 3 gives us eight, and we write 8. Then we go to the tens column, saying: 8 and 1 make 9 and 9 is written, then we say 5 is left. Total will be 598.

Observation. - If numbers in the same column make 10 when put together we write 0 in the units column and carry one into the tens column. In general when arriving at the sum, when numbers together make one or more tens, we write only the last digit, that is the units, and the tens that are considered as units are carried into the next column.

EXAMPLE:

First item	389
Second item	154
Third item	392
Total 935	

Beginning with the smallest item we say:

2 plus 4 make 6, plus 9 equals 15.

We write 5 under the units, and we carry one into the tens column saying:

1 plus 9 makes 10, plus 5 makes 15, plus eight equals 23.

We write 3 under the tens column and carry 2 into the hundreds column (these two equal 20 tens or two hundred units): then we continue: 2 plus 3 makes 5, plus 1 makes 6, plus 3 makes 9. The total will be 935.

Q. What is the sign that indicates that two numbers are being added?

A. This is indicated with a small cross + which means plus and it is put between the numbers being added.

So 3+4 tells us that the 3 has to be added to the 4 and we say 3 plus 4 equals 7, and the word *equals* is expressed by two short parallel lines like this: $3+4 = 7$.

Q. How do we **prove** the addition?

A. To **prove** the addition we add the items or rows up again but this time in reverse order, that is beginning from the bottom if we first began from the top, or from the top if we first began from the bottom. If the second total is equal to the first, the operation can be considered correct.

EXERCISES IN ADDITION.

1. An employer paid fr. 750 to rent a shop, plus 560 as an annual wage for two workers, plus 130 for an apprentice who showed special diligence in working for him. How much did he pay overall?

2. A carpenter spent fr. 1526 on planks, 2847 on beams, and bought tools for 235. How much did he spend overall?

3. A farmer spent fr. 300 on clothes for his family; 150 on wheat; 367 on corn. How much did he spend overall?

4. To keep his son at boarding school a father spends fr. 450 on boarding fees, fr. 215 on clothes, cleaning and repairs, fr. 97 on books and paper. How much does he spend overall?

III. - Subtraction

Q. What is **subtraction**?

A. **Subtraction** is an operation by which we take one number from another to know how much remains.

Q. What are the names usually given to numbers in subtraction?

A. The number to be subtracted from is called the **minuend**; the number that is used to do the subtraction is the **subtrahend**, while the number left over after this is called the **remainder** or **difference**.

Q. Can we do subtraction when the minuend is of a different species to the subtrahend?

A. Just as we cannot add two numbers of different species, nor can we subtract them.

Q. How do we do subtraction?

A. To do subtraction we write the subtrahend digits under the minuend digits in such a way that units are under units and tens under tens etc.: a line is drawn underneath and then beginning from the right, we subtract units from units, tens from tens, writing the difference below the line: the same is done with other digits, continuing left, until the operation is complete.

EXAMPLE:

A father pays 525 francs annual rent for his house, and has already paid 313; how much does he still have to pay?

Minuend	L.525
Subtrahend	L.313
Horiz. line & Rem.	L.212

To carry out this operation take 3 from 5 and we say: someone pays 3 out of 5 leaving 2 which we write under the line. Then we say 1 out of 2 is paid, leaving 1, and this is also written below the line. Then 3 out of 5 is paid; that leaves 2. The difference will be L. 212.

Q. What do we need to be careful of in subtraction?

A. To understand the various cases of subtraction we need to be careful that:

1. When the subtrahend digit is equal to the digit which corresponds to the minuend, we write 0 under the line:

2. When the subtrahend digit is greater than the digit corresponding to the minuend we take one unit from the next digit of the minuend on the left, and since this unit is a ten with respect to where it is being carried across to, it has a value of tens.

EXAMPLE:
A man buys a plot of land that costs L. 3405, of which he has already paid 1605. How much does he still have to pay?

Min.	3405
Subtr.	1605
Line & Rem.	1800

The operation is done this way:

5 from 5 leaves nothing, so we write 0 as the difference; 0 minus 0 leaves 0: we write 0 as the difference; 4 minus 6 or take 6 from 4 is taking too much, so we borrow a unit from 3 which, with respect to 4, being a ten, means 10 units, and added together makes 14; 14 minus 6 leaves 8; we write 8 as the difference. Now having taken 1 from 3, that leaves 2, so we say 2 minus 1 leaves 1. The remainder will be 1800.

Q. How do we do subtraction when there is one or more 0s in the minuend?

A. When there is a significant digit in the subtrahend and in the minuend we meet a 0, then the 0 counts as 10, and the first digit on the left decreases by one. If there is more than one 0 one after the after this rule is applied. The first 0 counts for 10, the others then count only as nine; but the first significant digit we meet on the left decreases by one.

EXAMPLE:
A baker had 3500 francs as capital; he has already spent fr. 1327 on grain. How much is still left?

Min	3500
Subtr	1327
Rem.	2173[6]

Q. How do we show that one number must be subtracted from another?

A. This is indicated by a horizontal line – (called minus), placed between the minuend and the subtrahend. So if we have to subtract 5 from 7, the subtraction is written 7–5=2, and we say seven minus five equals 2.

Q. How do we **prove** subtraction?

A. To **prove** subtraction we add the difference and the subtrahend. If the total is equal to the minuend then the operation is correct.

EXAMPLE:

A businessman has to provide 20550 bricks; he has already provided 12500. How many does he still have to provide?

Min.	20550
Subtr.	12500
Rem.	8050
Proof	20550

EXERCISES IN SUBTRACTION.

1. 1. A farmer has an annual income of lire 2650; he pays out 725 for a child at University; How much is left for the family?

2. 2. At the beginning of the year Rome had a population of about 290 000, and at the end of the year 8187 are registered as having died; how many are left?

3. 3. A man will live for 86 years, 11 months and 18 hours; how much life does he have left when he is 77 and 8 months, 16 hours?

IV. - Multiplication.

Q. What is **multiplication**?

A. **Multiplication** means repeating a number called the *multiplicand* as often as the number of units of another number called the *multiplier*. The *multiplicand* and *multiplier* are usually called *factors*. The larger factor is usually written first. The result of this operation is called the *product*. To learn multiplication we need to practise the following table:

2 times 2 makes 4	**4 times 4 makes 16**	**6 times 10 makes 60**
2 times 3 makes 6	**4 times 5 makes 20**	**7 times 7 makes 49**
2 times 4 makes 8	**4 times 6 makes 24**	**7 times 8 makes 56**
2 times 5 makes 10	**4 times 7 makes 28**	**7 times 9 makes 63**
2 times 6 makes 12	**4 times 8 makes 32**	**7 times 10 makes 70**
2 times 7 makes 14	**4 times 9 makes 36**	**8 times 8 makes 64**
2 times 8 makes 16	**4 times 10 makes 40**	**8 times 9 makes 72**
2 times 9 makes 18	**5 times 5 makes 25**	**8 times 10 makes 80**
2 times 10 makes 10	**5 times 6 makes 30**	**9 times 9 makes 81**
3 times 3 makes 9	**5 times 7 makes 35**	**9 times 10 makes 90**
3 times 4 makes 12	**5 times 8 makes 40**	**10 times 10 makes 100**
3 times 5 makes 15	**5 times 9 makes 45**	
3 times 6 makes 18	**5 times 10 makes 50**	
3 times 7 makes 21	**6 times 6 makes 36**	
3 times 8 makes 24	**6 times 7 make 42**	
3 times 9 makes 27	**6 times 8 make 48**	
3 times 10 makes 30	**6 times 9 make 54**	

One can also learn multiplication well by studying this other table called the Phythagorean table after its inventor, Pythagorus.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

It contains all the products whose factors are a single digit. To find these products we apply the following rule: We seek one of the factors in the first row at the top, and the other factor in the left hand column; the product is found at the intersection between the two. So for example I want the product of 5 multiplied by 8: I look for 5 in the top row and I see it is in the 5th column, then I look for 8 in the first column and I see it is found in the first box on the eighth row. Then I observe where the 5 in the column intersects with the eighth row and I see it is where the number 40 is: so we say 5 multiplied by 8=40

Q. How do we do multiplication?

A. Once the multiplier has been written under the multiplicand, we draw a line underneath, then take each digit in the multiplicand and 'times' it as many times as the units in the multiplier, and when the product is greater than ten, write only the units, while the tens are added to the product of the following digit.

EXAMPLE: -

What is the product of 453 multiplied by 3?

Multiplicand	453
Multiplier	3
Product	1359

Beginning from the right we go left with saying:3 times 3 is 9, so we write 9 in the product: 3 times 5 is 15, so we put down 5 and carry a ten to the following product; 3 times 4 is 12, plus 1 that we carried, and that gives 13 which we write as a complete number because there is nothing else to multiply. Our product is 1359.

Q. How do we do multiplication when there are more digits in the multiplier or there are zeros?

A. When there are two or more digits in the multiplier, then each of these is used to multiply the entire multiplicand, so there will be as many products as there are digits in the multiplier. Such products are called partial ones but be careful to write them in such a way that each partial product has its first digit under the corresponding digit of the multiplier. Then add the partial products together. When there are zeros in the multiplier, all we do is write a zero under them in the partial product and move on to the next digit.

EXAMPLE:

An agent in the countryside spends 280 francs a day on workers: how much will he spend in a year, or 365 days?

Multiplicand	365
Multiplier	280
1st Prod.	29200
2nd Prod.	730
Total Prod.	102200

We say 0 multiplied by 5 is 0; we write 0 in the product under the 0; 8 multiplied by 5 is 40, so we write 0 under the 8, and we carry 4 tens saying: 8 multiplied by 6 is 48, plus 4 that we carried over, makes 52; we write 2 and carry over 5 tens saying: 8 multiplied by 3 is 24 plus 5 that we carried over gives us 29, and we write down the whole 29. The first product will be 29200. We then move to the third digit in the multiplier saying: 2 multiplied by 5 is 10, and we write 0 in the second product but under the 2, and carry a ten saying: 2 multiplied by 6 is 12 plus one that we carried over, makes 13; we write 3 and carry a ten over saying: 2 multiplied by 3, is.6, plus one that we carried over, makes 7. The second product is 730. Adding these two products together we produce the total which is 102200.

Q. How do we do multiplication of a whole number by 10, 100, 1000 etc.?

A. It is enough to add a 0 to any number and it will be multiplied by ten, and add two zeros for a hundred, three 0s for a thousand and so on. So 3 multiplied by 10 is 30; 3 multiplied by 100 is 300. This happens because of the principle that a number acquires the value of 10 times larger for every digit that advances from right to left.

Q. When do we need to use multiplication?

A. When we know the value of a unit and we want the value of more of those units. So for example: we know that a metre of bread costs 8 lire, and we want to know how much 15 metres will cost. Or: we know that a day is equivalent to 24 hours and we want to know how many hours are equivalent to 6 days, meaning how many hours there are in 6 days.

Q. How do we show that two numbers are to be multiplied?

A. By putting the following sign, **x,** between them, made up of two lines which cross each other, and this is called the multiply sign. So if we have to multiply 3 by 4 we write 3 x 4 = 12, and we say: three multiplied by four equals twelve.

Q. How do we **prove** multiplication?

A. The simplest and easiest way to prove multiplication is to put the multiplicand where the multiplier is and repeat the multiplication. If the two products are equal we can believe that the multiplication is correct.

So for example: 12 x 20 = 240; by changing the order of the factors we have 20 x 12 = 240: the second product equals the first so it means the operation was correct.

EXERCISES IN MULTIPLICATION.

1. A young man spends 2 fr. on tobacco a week, fr. 5 on billiards, so how much would he save in a year if he abstained from these vices?

2. A mother buys 219 metres of bread at fr. 8 a metre: how much does she have to pay?

3. Every day has 24 hours, each hour 60 minutes, so how many hours and minutes are there in a day, a week, a month, a year, or 365 days?

4. How much do we have to pay for 85 hectolitres of wine at 23 francs a hectolitre?

V. - Division.[7]

Q. What do we mean by **division**?

A. By **division** we mean an operation by which we want to know how many times a number called a *divisor* is contained in another which we call a *dividend*. The resulting number is called a *quotient*. The *dividend* and *divisor* are also called the *terms* of division,

Q. How do we do division?

A. We write the dividend, separated from the divisor by a horizontal line and by another vertical line as seen in the figure that follows, then starting from the left of the dividend with as many digits as there are in the divisor we see how many times this goes into the digits we start with in the dividend. The result is written under the divisor and is called a quotient. This is multiplied by the divisor and the product is written under the digits in the dividend from which it is then subtracted. The remainder must always be less than the divisor, otherwise the digit in the quotient would be too small.

The following examples teach the way of doing division:

An employer wants to give fr. 92 to 4 of his boys as a New Year present; how much will each get?

Dividend	92	4	Divisor
	8	23	Quotient
	12		
	12		
	00		

The divisor is written to the right of the dividend as above, and we see how many times the divisor goes into the first digit of the dividend, and say: 4 into 9 goes twice so we write 2 in the quotient under the divisor; not to confuse the operation we should immediately put an apostrophe in front of the 9 to show we have already used it.

The same is done for all other digits. When we multiply the quotient 2 by the divisor 4, we get 8. This 8 is written under the 9 of the dividend and we do subtraction saying 9 minus 8 leaves 1. Then we continue: next to this one we write the other digit of the dividend below, which is 2, and we write it to the right of the 1 which, being a ten, gives us 12. Now we say: 4 into 12 goes 3 times; we put 3 in the quotient to the right of the 2 and multiplying 3 by the divisor 4 gives 12, which we write under the 12 of the dividend: and subtracting, we get 0. The quotient or amount each one gets is 23 francs.

[7] The material in Chapters V to XVII are enough for the programmes for 3rd Primary

Observation. - this operation is the rule when the divisor is contained in the first digit of the dividend.

Q. How do we do division when the divisor is not contained in the first digit of the dividend?

A. When the divisor cannot be contained in the first digit of the dividend, then we take two digits.

EXAMPLE:

Dividend	130	5	Divisor
	10	26	Quotient
	30		
	30		
	00		

We say: the divisor 5 does not go into the first digit of the dividend 1, therefore we take the following digit as well, that means we have 13. Now 5 into 13 goes 2 times; we write 2 in the quotient; 2 multiplied by 5 is 10, so we write 10 under the 13, and we do subtraction; then we continue as above.

Q. How do we do division when there are more digits in the divisor?

A. When there are more digits in the divisor we take as many digits in the dividend as there are in the divisor, and when the value of the digits in the divisor is greater than the digits in the dividend as an equal number, we take one more digit in the dividend.

EXAMPLE:

Dividend	450	25	Divisor
	25	18	Quotient
	200		
	200		
	000		

The two which is the first digit of the divisor goes two times into the first digit of the dividend; but the 5 which is the second digit in the divisor no longer goes twice into the 5 of the dividend; therefore we say: 2 into 4 goes once leaving 2 joined to 5 which makes 25. The 5 of the divisor easily goes once into 25: so we write one in the quotient. Then we multiply the quotient 1 by the divisor 25 and the product is 25, which we write under the 45. Subtracting we get 20 and next to it we bring down the last 0 from the dividend and get 200. Since the 25 cannot be divided by an equal number of digits we need to add one more; that means instead of 20 it becomes 200, saying: two goes into 2 in the dividend but 5 no longer goes into the following digits so we say: 2 into 20 goes 8 times; note however that 2 into 20 would go 10 times but we cannot go more than eight because we are looking for a quotient one digit at a time and not two, and not even 2 into 20 goes 9 times because it will not give a sufficient remainder which joined with zero can be divided by five nine times as well. Therefore we say that two into 20 goes 8 times with a remainder of four 4, which joined with 0 makes 40. Now 5 into 40 also goes 8 times and we write eight in the quotient, the 8 being multiplied by 25 to give 200. After subtraction there is a 000 remainder. The quotient is 18.

Observation. - If, during this operation, after having brought down a digit from the dividend it is still not enough to contain the divisor, we write zero in the quotient and bring down another digit from the same dividend.

Q. How do we divide a number ending in zeros by 10, 100, 1000 etc.?

A. If we want to divide by 10 we take away a zero and the number that remains will be the quotient. If we want to divide by 100 we take away two zeros, for 1000 we take off three.

So the number 20000 divided by 10 gives a quotient of 2000; divided by 100 it gives 200, divided by 1000 it gives 20. This occurs by the principle that a digit takes a value of being 10 times smaller as we move from left to right.

Q. When do we use division?

A. We use division:

1. When given the value of more units and the number of these units, we want to know what is the value of just one.
So for example 25 metres of bread cost lire 300, and we want to know what one metre costs.
2. When given the value of more units and the value of one we want to know how many units there are.
For example we have 450 lire to buy material that costs 9 lire a metre; we want to know how many metres we can buy.

Q. How do we show that one number is being divided by another?

A. This is indicated by a colon sign : (divided by) placed between the dividend and the divisor. So to indicate that we want to divide 9 by 3 we write 9:3=3 and we say nine divided by three is equal to three.

Q. How do we provide a division?

A. Proof of division is done by multiplying the quotient by the divisor and adding the remainder if there is one. If the sum equals the dividend the operation is done correctly.

Dividend	**441**	**7**	**Divisor**
	42	**63**	**Quotient**
	21	**7**	
	21	**441**	
	00		

EXAMPLE:

To prove this example we multiply the quotient 63 by the divisor 7 giving 411 which sum is equal to the dividend, so we are correct. If the division produces a remainder we need to add it to the product so it becomes equal to the dividend.

EXERCISES IN DIVISION.

1. A man who is moved by a true spirit of charity puts aside fr. 216 to give out to 9 poor families. How many fr. will each get?

2. A generous boy wants to give 500 nuts to 20 of his friends; how many will each get?

3. The father of a family has 2190 fr. annual income: how much can he spend a day so it lasts the whole year or 365 days?

VI - Decimal numbers

Q. What are **Decimal Numbers**?

A. They are numbers that express wholes or parts of units successively 10 times smaller.

Q. What do we call numbers that indicate parts which are tens of times smaller than a unit?

A. They are called **decimal fractions**.

Q. What do we have to take special note of in decimal numeration?

A. In decimal numeration we need to separate the fractions from whole units with a comma.

For example if I want to write 25 francs and 50 cents I will write 25, 50.

Q. The digits after the comma - what part of the unit do they express?

A. The first digit after the comma expresses **tenths** of the preceding unit, the second **hundredths**, the third **thousandths**, the fourth **ten thousandths**, the fifth **hundred thousandths**, the sixth **thousand thousandths** and so on.

If we have 42,356 metres. The number 42 expresses the units; the 3 because it is the first after the comma, expresses tenths of a metre; the 5 hundredths, the 6 thousandths. So to express the tenths one digit is enough after the comma, but two are needed for hundredths, three for thousandths, four for ten thousandths and so on.[8]

Q. How are decimal numbers written?

A. We begin by writing the whole number if there is one and if not we put a 0 to indicate that there are no units; then we put the comma; we then see how many digits are needed to express the kind of decimal fraction contained in the number proposed. If the decimal fraction considered as a whole number does not reach the same number of digits, we supply with zeros immediately after the comma.

If we want to write zero for the whole number and twenty five thousandths: to express the thousandths we need three digits, and to write twenty five we only need two; therefore immediately after the comma we put a zero thus: 0, 025.

[8] From this we see that we can add or take away zeros from the end of a decimal number without changing its value, because the first digit after the comma always indicates tenths, the second hundredths etc.

Q. How do we read decimal numbers?

A. We begin by reading the whole numbers, then we read the decimal fraction as if it was a whole number but we give all the decimal fraction the name of the final digit to the right.

So to read the number 5238, we say five (whole number), then we read the decimal part as a whole number saying: two hundred and thirty eight thousandths because the final digit on the right expresses thousandths.

EXERCISES IN DECIMAL NUMERATION.

1. Write the following numbers as digits: three wholes plus eight thousandths; zero metres and three hundred and twenty five thousandths of a metre; twenty thousand and four lire and three hundred and eight thousandths; fifty three hundredths.

2. Read the following numbers: 34,255; 0,06; 0,3045; 804,003006.

3. Say how many tenths are needed to make a whole; how many hundreds to make a tenth: how many thousandths to make a hundredth. How many hundredths are there in two wholes; how many thousandths in three tenths.

VII - Decimal addition.

Q. How do we add decimal numbers?

A. We do as do for whole numbers, being careful only to separate the wholes from the fractions with a comma; and when we move from the fractions column to the units, tens are carried over as usual without worrying whether they are whole numbers or fractions.

EXAMPLE:

A worker wants to give an exact account to his employer and notes expenses as follows:

Spent on cheese		fr 3,75
butter		fr 4,60
rice and vermicelli		fr 9,87
	Total	fr 18,22

So he says: 7 plus 5 is 12, then writes down 2 and continues: l plus 8 is 9, plus 6 is 15, plus 7 is 22; he writes 2 after which a comma to indicate fractions, then continues: 9 plus 2 carried over makes 11, plus 4 makes 15, plus 3 makes 18, total 18, 22.

EXERCISES.

1. A man who wants to use his wealth well writes out his will and for restoration of a church leaves L. 5500 and cent. 85. For the education of youth fr. 580 cent. 60 a year. For the poor fr. 434 cent. 45. How much does he leave altogether?

2. In a year a father saves fr. 825 cent. 90; by giving up a few amusements his son saves fr 226 cent. 32; the mother saves a further fr. 167 cent. 42 by being especially careful. How much has the entire family saved?

3. A mother buys 86, 17 metres of material to make sheets, 62,9 metres for shirts; 39,67 metres for towels. How many metres of material has she bought?

VIII. – Decimal subtraction

Q. How do we do subtraction with decimal numbers?

A. Subtraction of decimal numbers is done as for whole numbers, just making sure we separate the wholes from the decimal fractions in the remainder with a comma, which however must be in the same column as the minuend subtrahend.

Example:

I have to pay	341,28
I pay	141,17
Rem.	200,11

Observation.- If the subtrahend and minuend do not have an equal number of digits in the fraction, these are supplied with zeros.

Example:

	542,00	**added two zeros**
	240,75	
Rem.	**301,25**	

I am supposed to receive two lots of fr 542: I receive fr 240 cent. 75. How much do I still have to get?

EXERCISES.

1. 1. At the end of the year a worker should receive fr. 70, but because he lost time he receives fr, 15, 50. How much does he still have to take home?
2. 2. A worker owes the baker fr. 200, 20; he has paid fr. 55, 65, and now pays 118, 15. How much does he still owe?
3. 3. I bought 1425, 5 myriagrams of raw grapes; 217 are blemished to be thrown away, plus 131 to be eaten. How many myriagrams still remain?

IX.- Decimal multiplication

Q. How do we do decimal multiplication?

A. Decimal multiplication is done as for whole numbers, noting only:

1. When there are fractions, multiplication is done as if they were all whole numbers without taking account of the comma. Then in the product we separate as many digits as there are fraction digits in the two factors, with a comma;
2. To multiply a decimal number by ten, a hundred and a thousand just move the comma one, two or three digits from left to right.

Example:

I bought cloth:	120,50	metres
For each metre I paid	3,45	
Multiplication	60250	
	48200	
	36150	
Addition	415,7250	

The four digits are separated by a comma and the product is 415 fr. and 72 cent(*esimi*). The remainder would be 50 ten thousandths which are not counted in ordinary calculations.

Observation. – When there are not as many decimal digits in the product as need to be separated by a comma, we add to the left of the product as many 0s as are needed to complete the decimal digits, plus a zero for the whole numbers.

For example: if cheese is sold for fr. 0, 80 a kilogram how much will 0,07 cost?

Operation:

Multiplicand	0.07
Multiplier	0,80
Product	0,0560

560 ten thousandths would be the price corresponding to seven hundredths of kilograms. A 0 is added to complete the digits in the factors, and another to take the place of the unit.

2. How much does a piece of bread 25, 55 metres long cost at lire 10 a metre? To solve the problem we only need to move the comma one place to the right: the product will be L. 255, 5.

EXERCISES.

1. How much would 343, 68 kilograms of bread cost at 0,45 a loaf?
2. A young man received fr, 1, 60 every Sunday from his father as pocket money; being a moderate sort of person he kept it all to buy clothes and give some to the poor; how much would he save in a year assuming 52 Sundays in a year?
3. Michael, a good lad, gets L. 0, 05 a day to buy fruit: every month he gave 0, 50 in alms, the rest he spent buying good books. How much did he give in alms? How much was left to spend on good books?

X. - Decimal division.

Q. How do we do decimal division?

A. Decimal division is done as we do with whole numbers, noting however the following:

1. When the dividend and divisor have an equal number of digits after the comma, ignore these and do the operation as if it was a whole number, and the quotient will then be a whole number.
2. When the dividend or the divisor has an unequal number of digits in the fraction, they can be made equal by adding 0s then continue as above.
3. When a decimal number has to be divided by 10, 100, 1000 etc., it is enough to shift the comma one, two, three digits from right to left,

EXAMPLE FOR 1st CASE:
I spent fr. 678 cent. 75 on 45 and 25 hundredths metres of material; How much does each metre cost me?

Dividend	67875	4525	Divisor
		15	Quotient

Observation. - In the example given do the same as if dividing 67875 by 4525; the 15 (15 francs) is the price of each metre.

EXAMPLE FOR 2nd CASE:
I paid fr. 115 cent. 50 for 5, 5 myriagrams of coffee; how much does each myriagram cost? (5 tenths of a myriagram is 5 kilograms).

Dividend	11550	550	Divisor, to which we add a zero
		21	Quotient

Observation. - We add a zero so the fractional digits in the divisor are equal to those in the dividend, and the division is done as usual giving us a quotient of fr. 21 which is the price of each myriagram.

N. B. If there are only decimal fractions in the dividend the division can be done without adding zeros to the divisor; one just has to be careful to put a comma in the quotient when beginning to take a decimal digit from the dividend, so for example 7, 26:3.

Dividend	7,26	3 Divisor
	6	2,42
	12	
	12	
	006	
	6	
	0	

EXAMPLE FOR 3rd CASE.

Charles has L. 343, 25 to give out to 100 poor people. How much will each get?

Dividend 343, 25, divisor 100; Quotient is L. 3, 4325 obtained by simply shifting the comma two digits to the left.

Q. How is division done when the dividend is less than the divisor?

A. The operation is done as usual, putting a zero before the quotient to indicate that the digits do not express whole numbers, and the dividend is increased by a zero to the right if that is enough, otherwise add two, three etc. Don't forget to put one, two etc. zeros after the comma in the quotient.

EXAMPLE: -

How do we divide 6 francs amongst 15 people?

60	15 Divisor
	0,4

We add one 0 to the dividend; the added 0 in the dividend makes the number ten times greater, but the value is always the same because these new parts are ten times smaller than before: meaning that the units with a 0 added become tenths; by adding another they become hundredths. Therefore in the dividend instead of 60 tenths we have 600 hundredths and instead of 4 tenths in the quotient we have 40 cent(*esimi*).

Q. What do we have to do when there is a remainder less than the divisor when we complete the operation?

A. We add a 0 to this remainder then we have tenths. By adding another 0, we have hundredths and we continue with the division. But when we add a 0 to get tenths or hundredths then we need to immediately put a comma in the quotient to separate the wholes from the fractions.

EXAMPLE: - 20 francs are to be shared amongst 3 workers

Dividend	**20**	**3 Divisor**
subtract	**18**	**6,66**
to convert to tenths we add a 0	**20**	
subtract	**18**	
to convert to hundredths, add 0	**20**	
subtract	**18**	
	2	

The quotient is 6, 66. The remainder 2 (hundredths) could be converted to thousandths by adding a 0 and continuing the division, but thousandths are usually ignored in ordinary calculations.

EXERCISES.

1. A baker sells 800 myriagrams of bread per week; how many does he sell each day?
2. A miller charges fr. 720, 75 for 28, 19 hectolitres of grain; how much for one hectolitre?
3. A merchant has fr. 2345 in his till after selling 200, 4 metres of bread; how much did he charge per metre?

The Metric Decimal System

XI. - General idea of this system.

Q. What do we mean by the **metric decimal system**?

A. By the **metric decimal system** we mean the set of all weights and measures using a *metre* as the base. We say decimal because it follows the decimal system of counting (numeration).

Q. What is a **metre** and how long is it?

A. A **metre** is one ten-millionth of the length of the earth's meridian along a quadrant, which is to do with the circumference of the earth.

That means that if we could wrap a thread around the earth and it could be divided into forty million equal parts, a part would be a metre in length.

Q. What does the word metre mean?

A. The word metre means *measure.*

Q. Why prefer this system to the one already in use?

A. Because it makes calculations easier: but even more so, since the metre is the same length anywhere around the world we avoid the variety of weights and measures we find even in the same State at times, even in the same Province! This diversity of weights and measures is open to errors and to all kinds of unfair manipulation. This can easily be avoided in places where the metric system is in use.

XII. - Basic units.

Q. Which are the basic units of the metric decimal system?

A. There are six basic units in this system:

The **metre** is used for length measurement.
The **square metre** is used for surfaces.
The **cubic metre** is used for volume.
The **litre** is used for quantity (capacity) such as wine, water, grain, corn and the like.
The **gram** is used for weights
The **franc** or **lira** are used for money.

Q. What measures use the **metre**?

A. The **metre** is used for all measures of **length** such as cloth, bread, roads and the like.

Q. Is the metre used to measure a floor, walls of a house, fields, meadows, vineyards?

A. To measure surfaces we use the **square metre** which is a surface with four sides, each a metre long. But since this measure would be too small for farm plots, in place of the square metre a **square decametre** is used, which is a surface with four sides each of which is ten metres long.

Q. What name do we give to a square decametre?

A. The square decametre is called an **ara** [in English this term is not used but it is equivalent to 100 square metres].

Q. What is a **cubic metre** and what is it used for?

A. The **cubic metre or stero** [this latter term is not used in English], is a body a metre high, long and wide. But the stero has a form which is different from the cubic metre, made like a die so it can be used for hay, straw, wood, gravel and the like.

Q. What is a **litre**?

A. The **litre** is a cubic decimetre. To get an idea of this let us imagine a linear metre divided into ten equal parts, and we have a decimetre or the tenth part of a metre. Now a cubic decimetre, or a container which is a decimetre long, wide and high is the capacity of a litre. It is used for measuring **capacity**, meaning for liquids like oil, wine, beer etc. and for dry material like wheat, rice, chestnuts, cheeses, beans etc.

Q. What do we mean by **gram**?

A. A **gram** is the weight of distilled water contained in a cubic centimetre. If we take a linear metre and divide it into a hundred equal parts each of these parts is a centimetre. A cubic centimetre is a container which is a centimetre long, wide, high. The gram is used for measuring **weight**.

Q. What do we mean by the **franc or new lira**?

A. We mean a silver coin which weighs five grams. It is used for measures of **value**, that is to determine the price of an object, work etc.

Q. How can we show that all measure derive from the metre?

A. The metre, being the basis of all decimal measures means that all other derive from it.

The *ara or square decametre* is a square whose sides are ten metres long.
The *stero or cubic metre* is equal to a die with metre length edges: that means a metre long, wide, and deep.
The *litre* comes from metre being the capacity of a cubic decimetre.
The *gram* also comes from the metre since it is the weight of a cubic centimetre of pure, distilled water.
The *franc* also comes from the metre since it weighs five grams.

XIII. - Decimal multiples and submultiples.

Q. What is meant by a **decimal multiple**?

A. By a **decimal multiple** we mean one of the units indicated below made ten times greater.

For example. 1 multiplied by ten makes 10. These ten are called *Deca-*: 10 multiplied by ten makes 100, called a *Hecto-*.

Q. What do we mean by submultiple?

A. By submultiple we mean the unit made ten times smaller.

E.g. 1 divided by 10 makes a tenth of the unit, called a *Deci-*.

Q. How many multiples are there?

A. Multiples, or rather the terms used to express an increase are four in number, expressed in the following Greek words:

Deca which means ten units;
Hecto which means a hundred;
Kilo which means a thousand;
Myria which means ten thousand.

Q. How many submultiples are there?

A. Submultiples, or rather the terms used to express parts of units, are three in number:

deci which means the tenth part of a unit,
centi, the hundredth;
milli, the thousandth.

Q. What is the difference between *deca* and *deci*?

A. *Deca* means ten units, *deci*, the tenth part of the same unit.

Q. How do we apply multiples to basic units?

A. If to a *Deca, Hecto, Kilo, Myria*, I add a metre, I have a *Decametre, Hectometre, Heliometre, Myriametre*. We do the same with other units.

Q. How do we apply submultiples?

A. If I add *metre* to the terms *deci, centi, milli*, I get *decimetre, centimetre, millimetre*, or the tenth, hundredth, thousandth part of a metre.

The following can help explain what was said above.

Written term	**Term in digits**	**Tern in decimals**
Unit	1	Unit
Ten	10	Deca
Hundred	100	Hecto
Thousand	1000	Kilo
Tens of thousands	10000	Myria
Hundreds of thousands	100000	Deca-Myria[9]
Million	1000000	Hecto-Myria

When it is a case of weights the deca-myria is called a *quintal* and the hectomyria is usually called a *ton*.

From this we see that a digit becomes ten times greater to the extent that it moves to the left. On the contrary each time a digit moves towards the right it becomes ten times smaller, so:

Unit	1	Unit
Tenth	0,1	*Deci* or tenth part of unit
Hundredth	00,1	*Centi,* or hundredth part of unit
Thousandth	000,1	*Milli*, or thousand ...
Ten thousandth	0000,1	*Decimilli*, or ten thousandth....
Hundred thousandth	00000,1	*Centimilli,* or hundred thousandth—
Millionth	000000,1	*Millimilli*, or thousand thousandth....

XIV. - Reading and Writing numbers expressing Metric Decimal Measures.

Q. Are the numbers expressing decimal measures written and read according to the rules of decimal numbers?

A. Yes, as a general rule they are written and read according to the rules for decimal numbers; we just need to note:

1. Sometimes we take as a unit of measure what is a multiple of the true measure and in this case behind this multiple we immediately put the comma, and the numbers that follow would be considered as its submultiples or decimal fractions. So although the true unit of measure for weights is the gram, nevertheless the kilogram is often seen as the measure. If for example we have to write four kilograms and twenty eight decagrams. In this number kilograms are regarded as the unit of measure so after 4 we write the comma, and then after it the other part of the decimal number thus: 4, 28.

2. We also note that for surface measures each multiple or submultiple or unit is worth a hundred times of the multiple or submultiple of the immediately lower unit. Thus a square decametre is equivalent to a hundred square metres, a square metre is equivalent to a hundred square decimetres, a square decimetre is equivalent to a hundred square centimetres;[10] so to write this we need two digits for each submultiple, one for the tens and the other for the units and we add zeros when units or tens are missing. So to write two square metres and three decimetres we write 2, 03, adding the zero in to supply for the missing tens in the square decimetres. If we need to write four hundred square metres and two hundred and sixty square centimetres, we write 400, 0260, where the first zero after the comma supplies for tens of square decimetres, and another zero supplies for the units of square centimetres. If we then read such numbers, we divide the digits to the right of the comma in twos from left to right; then we read the entire decimal fraction as a whole number calling it by the name of the last item on the right. So to read the number *Hectm.* q. 28, 5626; since the unit of measure here is square hectometres, the first two digits after the comma will be square decametres and the last two square metres, and we say 28 hectometres and five thousand six hundred and twenty six square metres.

[10] In fact if we divide a square metre into so many squares a decimetre in length and breadth, we find that there are a hundred of them (square decimetres) in a square metre

3. Finally, we should note that for cubic measures each unit, or multiple or submultiple is equivalent to a thousand times the unit,[11] or multiple or submultiple immediately inferior to it; therefore we need three digits to express tenths, that is units, tens and hundreds of tenths, three digits to express hundredths, etc. and we add zeros for units, tens and hundreds missing; to read such numbers we divide the digits in the fraction part in threes from left to right, the first three after the comma expressing cubic tenths, the other three cubic hundredths; and by reading it as a whole number the fraction takes the name of the last item on the right. To write the number four hundred cubic metres and thirty six cubic decimetres we write 4, 036 putting the zero because hundreds of cubic decimetres is missing. To read the number *m. c.* 8, 367608 we begin dividing the digits of the decimal fraction in threes:[12] so we find three digits for cubic decimetres, and three for cubic centimetres, and we say 8 cubic metres and three hundred and sixty seven thousands six hundred and eight cubic centimetres.

Q. Does each of the basic units have all the multiples and submultiplesi?

A. The metre, litres, gram have all four multiples, and all three submultiples. But the *ara* has only one multiple which is the *hectare* (100 *are*) and only one submultiple, which is the *centiara* (hundredth part of an *ara*). The *stero* only has a *decastero* and DECISTERO.

Q. What abbreviations are sued in the metric decimal system?

A. As a general rule the unit of measure is indicated by its initial letter in lowercase. So to write 6 metres, 15 grams, etc. simply write m. 6, g. 15. To express the multiples, on the left of this letter write in upper case the initial letter of the multiple; then we write the lower case initial letter of the submultiple when a submultiple has to be written. Thus to abbreviate the expression: two decalitres we write Dl. 2; to abbreviate 44 centigrams we can write Cg. 44. So also for Kg. 36, 75; Em. 5, 26, Dl. 7, 5 we read 36 kilograms and 75 decagrams; 5 hectometres and 26 metres; 7 decalitres and 5 litres.

[11] In fact by dividing a cubic centimetre into so many little die a decimetre long, wide and deep, we find there are a thousand of these in a cubic metre, that is, one thousand cubic decimetres.

[12] We need to note that this separation into groups of decimal digits is done from left to right and not right to left like when reading a whole number. The first 3 digits represent cubic decimentres, the next 3 cubic centimetres, etc.

XV. - Ordinary Fractions.

Q. What do we mean by **ordinary fractions**?

A. **Ordinary fractions** are those that express the parts of the unit in whatever way it is divided. Live five eighths of a page, three quarters of the world. half a nut.

Q. With what numbers do we usually express a fraction?

A. A fraction is normally expressed with two numbers called a **numerator** and a **denominator**. The denominator indicates how many parts the unit is divided into, the numerator indicates how many of these parts we are dealing with.

Q. How do we say the numerator and denominator?

A. The numerator is said by saying the number it represents as it is written; we say *one, two, three, ten, twenty five* etc. In the denominator the numbers *two, three, four*, etc. up to ten are called *half, thirds, quarters, fifths, sixths, sevenths, eighths, ninths, tenths*; after ten we continue similarly. [in Italian we add *-esimi*, so in Italian we would say: *tre undicesimi, quattordici quarantacinquesimi*].

Q. How do we write fractions?

A. By putting the numerator above the denominator with a horizontal or oblique line separating them as in $\frac{3}{4}$ or 3/4.

Q. How do we subdivide ordinary fractions?

A. Fractions are subdivided into **proper** and **improper**. **Proper fractions** are those that express a **lesser** number of the unit, such as $\frac{1}{2}$, $\frac{4}{7}$; the numerator is less than the denominator. **Improper fractions** are those having a numerator greater than the denominator and they contain not only parts of the unit or just whole units but units and parts of a unit, such as $\frac{17}{5}$, $\frac{26}{6}$. They are called **apparent fractions** if both **terms are equal**, or have a numerator that is a multiple of the denominator,[13] that is twice or three times etc.; as for $\frac{3}{3}$ $\frac{8}{2}$. Although these fractions are written in the form of a fraction they are equivalent to whole units. In fact $\frac{3}{3}$ equals one unit; $\frac{8}{2}$ equals four units. A mixed number is one made up of units and fractions.

So for example $3\frac{2}{5}$; $5\frac{13}{15}$ are mixed numbers.

[13] A number is called a multiple of another when it contains it an exact number of times. So 9 is a multiple of 3, 13 of 3 and 4.

Q. How do we split an improper fraction, that is, how can we separate the wholes from the fractional parts into an improper fraction?

A. By dividing the numerator by the denominator. The quotient expresses the wholes, the remainder will be the numerator of the fraction part, while the divisor continues being the denominator.

So to extract the units from $\frac{17}{5}$ we divide the 17 by 5

$$\begin{array}{r|l} 17 & 5 \\ \hline 15 & 3\frac{2}{5} \\ \hline 2 & \end{array}$$

The quotient 3 indicates the units, the remainder 2 will be the numerator and the divisor 5 the denominator of the new fraction, giving us $\frac{17}{5} = \frac{32}{5}$

Q. How do we convert a whole number into a fraction, that is into thirds, fourths, elevenths etc.

A. By multiplying the whole number by the denominator we want to give it, that is, if we want to convert into thirds we multiply by 3; for fourths we multiply by 4, into elevenths by 11 etc., and we give this number to the product as the denominator. So if we want to covert 5 wholes into sixths, we multiply 5 by 6 and the six is also used as the denominator, so we get $5 = \frac{30}{6}$.

Q. How do we convert a number made up of wholes and fractions into a single fraction?

A. By multiplying the denominator by the wholes and adding the numerator to the product, leaving the denominator the same.

So to convert 3 wholes and 2 fifths into a fraction, we multiply the 5 by 3, then to 15 which is the product we add the numerator 2, and this gives us $3\frac{2}{5} = \frac{17}{5}$.

Q. What change does a fraction undergo if we **multiply** only one of its terms?

A. If we **multiply** only its **numerator**, the fraction is **multiplied**, so given the fraction $\frac{2}{3}$ if I multiply the numerator two by four, I get $\frac{8}{3}$ which is a fraction 4 times greater than $\frac{2}{3}$; on the other hand if I multiply only the denominator, the fraction is divided. So in the above fraction 2/3, if I multiply the denominator 3 by 4 I get $\frac{2}{12}$ that is four times smaller than $\frac{2}{3}$ since the denominator 12 indicates that the unit was divided 4 times smaller.

Q. What change happens to a fraction when only one term is **divided**?

A. It changes according to the term that is **divided**. By dividing the **numerator** the fraction is **divided**, so for $\frac{6}{8}$ by dividing the numerator 6 by 2 I get $\frac{3}{8}$, a fraction two times smaller than $\frac{6}{8}$. On the other hand by dividing its **denominator** the fraction is multiplied, so for $\frac{6}{8}$ by dividing the denominator 8 by 2 I get $\frac{6}{4}$ a fraction which is twice as large as $\frac{6}{8}$, since the parts into which the unit is divided become larger; in fact fourths are double the size of eighths.

Q. What change does a fraction undergo by **multiplying or dividing** the two terms by the same number?

A. The fraction does **not change** value.

So for example by multiplying by 2 the two terms of the fraction $\frac{1}{2}$ we get $\frac{2}{4}$ which is perfectly equal to a half: so also by dividing the terms of the fraction $\frac{4}{8}$ by four we get $\frac{1}{2}$ which is perfectly equal to$\frac{4}{8}$. From this we see that by multiplying or dividing the terms of a fraction by the same number the fraction does not change value but is changed into an equivalent one.

Q. Can we not convert an **ordinary fraction** into a **decimal fraction**?

A. We can convert an ordinary fraction into a decimal fraction by dividing the numerator by the denominator. When the denominator is not contained in the numerator we put a zero in the quotient followed by a comma and we also a zero to the dividend then do the division following the rules given earlier; the digits we get in the quotient will be decimal fractions.

Convert the ordinary fraction 3/4 into a decimal fraction.

30	4
28	0,75
20	
20	$\frac{3}{4} = 0{,}75$
00	

Divide 3 by 4: since the 4 is not contained in 3 we put a zero in the quotient with a comma, and add a 0 to the dividend. Continuing the division we get 0,75 as the quotient. Doing this we get many decimal fractions equivalent to ordinary ones and other times we cannot get perfectly equivalent decimal fractions but ones with approximate value the more we continue the division.

§.1. Converting fractions to minimal terms.[14]

Q. What does it mean to convert fractions to **minimal terms**?

A. It means making the terms as small as they can be or reducing them to their simplest expression.

Q. How do you reduce a fraction to its simplest expression?

A. To reduce a fraction to its simplest expression, begin by seeing if its terms are divisible by the same number, and then divide by this number as far as we can; or divide it by another number as far as we can, until the terms no longer have a divisor that can divide both of them, that is a common divisor.

[14] The following paragraphs on fractions are not part of the Primary course. They are added here to help anyone who wishes to complete the study of this part of arithmetic

FOR EXAMPLE;

$\frac{44}{66} = \frac{22}{33} = \frac{2}{3}$
$\frac{54}{96} = \frac{27}{48} = \frac{9}{1}$
$\frac{46}{72} = \frac{23}{36}$

Q. What do we call fractions hose terms have no common divisor?

A. They are called **irreducible**.

EXAMPLE:
The fraction $\frac{14}{57}$ is irreducible because the fraction $\frac{14}{57}$ cannot be expressed in smaller digits.

Q. Is there any other way to reduce fractions to minimal terms?

A. When at first sight we cannot find a common divisor for the two terms we go looking for the **largest common divisor**.

Q. What is the **largest common divisor**?

A. The LCD is the largest number that exactly divides the two terms of a fraction.

Q. What do we do to find the LCD of a fraction?

A. If we have a fraction we divide the larger term by the smaller one, write the quotient above the divisor and if there is a remainder, it becomes the divisor in the first divisor, so it is written on the right. The new quotient is written above the new divisor, and the remainder becomes the divisor of this second divisor; continue this way till we find a divisor that divides its dividend exactly. This number is the LCD.

EXAMPLE:

$\frac{143}{637}$ $\frac{637}{65}$

4	2	5
143	65	13
13	0	

Q. What is the LCD. for?

A. Since it exactly divides the two terms of a fraction it is used to quickly reduce a fraction to its minimum terms. In fact in the preceding example 143:13 = 11 and 637:113 = 49 therefore $\frac{143}{637} = \frac{11}{49}$

§ 2. Reducing fractions to a common denominator.

Q. What does it mean to reduce fractions to a **common denominator**?

A. It means to do things in such a way that two or more fractions end up with the same denominator without changing the value.

Q. How do we reduce fractions to a common denominator?

A. We multiply the two terms of each fraction by the product of the denominators of all the others.

EXAMPLE:

$\frac{2}{3}, \frac{4}{5}, \frac{3}{4} = \frac{40}{60}, \frac{48}{60}, \frac{45}{60}$

Q. On what principle is this reduction to the common denominator based?

A. On the principle that by multiplying the two terms of a fraction by the same number, this fraction does not change value; in fact we do nothing else but multiply the two terms in each one by the same number, that is, by the product of the denominators of the others.

Q. Is there any other way of reducing to the common denominator?

A. It could sometimes happen that if we have other fractions to reduce, we might find one whose denominator is a multiple of the denominators in all the others; in this case this denominator is the common denominator and to get the numerator for each fraction we divide the common denominator by the denominator of each of the fractions. The quotient then is multiplied by the numerator of the corresponding fraction. The product will be its numerator.

EXAMPLE:

1	***5***	***10***	***8***	***20***	***2***
$\frac{36}{40}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{14}{20}$
$\frac{36}{40}$	$\frac{35}{40}$	$\frac{30}{40}$	$\frac{16}{40}$	$\frac{20}{40}$	$\frac{28}{40}$

Q. What change is there in a fraction if we add or subtract the same number from its terms?

A. If it is a proper fraction it increases or decreases in value according to what is added or taken away; if it is an improper fraction then it diminishes by adding, and increases by taking away the same amount from the two terms.

§ 3. Addition of Fractions:

Q. How many cases of adding fractions are there?

A. Two cases:

1. Addition of proper and improper fractions;
2. Mixed fractions or fractional numbers.

Q. How do we do addition in the first case?

A. If two 0s plus other fractions are proposed for addition they must first be reduced to the same denominator if such is not already the case, then we immediately add the numerators and give the total the common denominator. If the resulting fraction is an improper one, whole numbers can be extracted in the way described above.

EXAMPLE:
$\frac{2}{3}+\frac{4}{5}+\frac{1}{2}=\frac{20}{30}+\frac{24}{30}+\frac{15}{30}=\frac{59}{30}=1\ \frac{29}{30}$

Q. How do we do addition in the second case?

A. Firstly we add the proper fractions in the way shown, and if the resulting fraction is improper, extract the wholes; then add all the whole numbers.

EXAMPLE:
$2+\frac{1}{3}+1\ \frac{2}{6}+8+\frac{9}{12}=(2+1+8)+(\frac{1}{3}+\frac{2}{6}+\frac{9}{12})=$
$11+\frac{17}{12}=12\frac{5}{12}$

EXERCISES.

1. Do the following additions: $\frac{35}{75}+\frac{15}{75}+\frac{25}{75}$; $\frac{12}{28}+\frac{9}{14}$; $\frac{7}{13}+\frac{3}{7}$; $\frac{1}{12}+\frac{3}{4}+\frac{2}{3}$
2. A poor man, happy with the alms he has received, adds them up. In the morning he received $\frac{9}{20}$ plus $\frac{2}{10}$ of a lira. In the evening $\frac{1}{5}$ plus $\frac{10}{25}$ of a lira. How much did he receive in the day?
3. 5 and $\frac{3}{5}$ litres of water are poured into a container, then 4 and $\frac{11}{12}$ litres and finally it is topped up with 3 and $\frac{3}{2}$ litres. How much water is there in the container?

§.4 Subtracting fractions.

Q. How many cases of subtracting fractions are there?

A. There are three cases:

1. Subtracting a fraction from a whole;
2. Subtracting one simple fraction from another simple fraction;
3. Subtracting mixed fractions.

Q. How do we do subtraction in the first case?

A. In the first case whole numbers are converted into fractions with the denominator of the given fraction, then subtraction of numerators is done giving the same denominator to the remainder, so we get a fraction from which we can again extract the wholes.

EXAMPLE:
$3 - \frac{2}{3} = \frac{9}{3} - \frac{2}{3} = \frac{7}{3} = 2 + \frac{1}{3}$

Q. How do we do subtraction in the second case, that is, when do we have to take a simple fraction from another simple fraction?

A. Both fractions have to be reduced to the same denominator if not already the case, then do subtraction of the numerators giving the same denominator to the remainder.

EXAMPLE:
$\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

Q. How do we do subtraction of fractions in the third case, that is, when there are two fractional numbers?

A. The fractional numbers are converted into improper fractions, then reduced to the same denominator, and subtraction is then done in the way that was shown. If the remainder is an improper fraction, the wholes are extracted.

EXAMPLE:
$3\frac{2}{7} - 8\frac{2}{9} = \frac{23}{7} - \frac{26}{9} = \frac{207}{63} - \frac{182}{63} = \frac{25}{63}$

EXERCISES.

1. Do the following subtractions: $\frac{25}{36} - \frac{21}{36}$; $\frac{4}{5} - \frac{3}{4}$; $\frac{35}{50} - \frac{7}{15}$; $5\frac{2}{7} - 3\frac{2}{3}$
2. A merchant sold $\frac{3}{7}$ of a piece of bread; how much of the piece remains?
3. A traveller has completed $\frac{1}{3}$ plus $\frac{1}{8}$ plus $\frac{2}{5}$ of his journey; how much is still left?
4. You buy m. $25\frac{1}{2}$ of bread. You sell m. $7\frac{3}{4}$ How much remains?

§.5. Multiplication of Fractions.

Q. How many cases of multiplication of fractions are there?

A. Three cases:

1. Multiplication of a whole by a fraction and vice versa;
2. Multiplication of simple fractions;
3. Multiplication of fractional numbers.

Q. How do we do multiplication in the first case?

A. We multiply the whole by the numerator and give the same denominator to the product.

EXAMPLE:
3 x $\frac{2}{7} = \frac{6}{7}$

Q. How do we do multiplication in the second case?

A. Multiplication in the second case is done by multiplying the numerators, then the denominators.

EXAMPLE:
$\frac{2}{5}x\frac{3}{7} = \frac{6}{35}$

Q. How do we do multiplication in the third case?

A. To multiply two fractional numbers first they have to be converted to improper fractions then carry out the multiplication as earlier indicated.

EXAMPLE:
$2\frac{1}{2}$ x $5\frac{2}{3} = \frac{5}{2}$ x $\frac{17}{3} = \frac{85}{6} = 14 + \frac{1}{6}$

EXERCISES.

1. Do the following multiplications: $\frac{35}{60}$ x 10; $\frac{25}{37}$ x 9; $\frac{3}{4}$ x $\frac{2}{5}$ x $\frac{3}{7}$; $\frac{1}{2}x5 + \frac{4}{7}$. Find $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 300 lire.
2. Henry knows he can complete $\frac{1}{2}$ of his trip in an hour; after $\frac{3}{4}$ of an hour how much has he done?
3. What is the number of which $\frac{5}{6}$ of $\frac{8}{9}$ make L. 20?

§ 6. Division of Fractions.

Q. How many cases are there of division of fractions?

A. Three:

1. Division of a whole number by a fraction and vice versa;
2. Division of a simple fraction by another simple fraction;
3. Division of fractions when there is a mixed fraction involved.

Q. How do we do division in the first case?

A. To divide a whole by a fraction put the whole over a denominator of one, then turn the fraction upside down and multiply them.

EXAMPLE:

$3\colon \frac{2}{5} = \frac{3}{1}$ x $\frac{5}{2} = \frac{15}{2} = 7 + \frac{1}{2}$

Vice versa, to divide a fraction by a whole, multiply the denominator by the whole and leave the numerator the same.

EXAMPLE:

$\frac{3}{4}\colon 8 = \frac{3}{4x8} = \frac{3}{32}$

Q. How do we do division in the second case?

A. To divide one fraction by another we need to turn the divisor fraction upside down, then do multiplication.

EXAMPLE:

$\frac{3}{7}\colon\frac{2}{4} = \frac{3}{7}$ x $\frac{4}{2} = \frac{12}{14} = \frac{6}{7}$

Q. How do we do division in the third case?

A. To do division where there are mixed fractions, firstly we need to convert the mixed fractions into improper ones then turn the divisor fraction upside down and do multiplication.

EXAMPLE:

$3\frac{1}{2}\colon 2\frac{1}{4} = \frac{7}{2}\colon\frac{9}{4} = \frac{7}{2}$ x $\frac{4}{9} = \frac{28}{18} = \frac{14}{9} = 1\frac{5}{9}$

EXERCISES

1. Do the following divisions: $\frac{75}{120}\colon 6$; $\frac{5}{4}\colon 3$; $8\colon\frac{4}{5}$; $3\colon\frac{5}{6}$; $\frac{7}{2}\colon\frac{5}{6}$; $\frac{8}{15}\colon\frac{6}{15}$; $\frac{125}{720}\colon\frac{3}{4}$
2. In $\frac{3}{4}$ hour a courier covers 6 kilometres; how long does it take him to do one kilometre?
3. A man sells his house for L. 4568 but this is an increase of $\frac{3}{20}$ of what it cost him. How much did it cost him?
4. In $\frac{3}{4}$ hour we can do $\frac{3}{8}$ of our work. How much would we do in an hour?

§ 7. Complex numbers and their conversion into ordinary and decimal fractions and vice versa.

Q. What are complex numbers?

A. Complex numbers are those made up of more parts which refer respectively to further subdivisions of the same unit. For example 4 *trabucchi* [*trabucco*: length measure, not used in English; cf. example below], 2 feet, 3 inches; 4 *rubbi* [*rubbio* was used in Papal States - area measure], 7 pounds, 8 inches; 2 lire, 11 *soldi*, 7 *denari* - are all complex numbers.

Q. What is the first operation we have to do to convert a complex number to a decimal fraction?

A. The first operation is to convert the complex number into an ordinary fraction of the main unit.

Q. What does it mean converting a complex number into an ordinary fraction of the main unit?

A. It means converting the complex number to its last subdivision and giving the resulting number the unit of the same kind converted to the final subdivision of the complex number as a denominator.

EXAMPLE:

Trab. Feet. Inches

3. 4. 9.

First we convert the *trabucchi* into feet. Each *trabucco* is equal to 6 feet, so 3 are equal to 18 feet. We add the 4 feet we have to these 18, giving us 22. Now the 22 feet have to be converted to inches. Each foot is 12 inches, so 22 feet is 264 inches, to which we add the 9 that we have, making 273 inches. This number will be the numerator. To find the denominator we convert 1 *trabucco* into feet, which gives us 6 feet. The 6 feet into inches gives us 72 inches. This give us the complex number equal to the ordinary fraction of a *trabucco*.

Tr. 3, ft. 4, in. $9 = \frac{273}{72}$

Q. What more do we have to do to convert a decimal fraction into a complex number?

A. To convert a decimal fraction into a complex number, we need to convert the decimal fraction into an ordinary fraction, then divide the numerator by the denominator, and when there are no further digits to bring down, multiply the remainder, if there is one, by the first subdivision of the complex number we want to convert to, and again divide the product by the same dividend. If there is still something over, we multiply it by the second subdivision and so on. In the quotient we need to separate the whole numbers from the units of the first subdivision, and these units from the second subdivision etc.

347	100
300	3,9,4
47	
20	
940	
900	
40	
12	
80	
40	
480	
400	
80	

So if I have to convert L. 3,47 into a complex number, first I reduce this decimal number into an ordinary fraction and I get $\frac{347}{100}$. That done I divide the numerator by the denominator and get 3 as the quotient, which is 3 lire, with 47 carried over. Now I multiply 47 by the first subdivision of the lira, which is 20 *soldi*, and I divide the product 940 by 100. With a comma after the 3 in the quotient, I find that the 100 goes 9 times into 940, so I put the 9 that will be 9 *soldi*, in the quotient, and I carry over the 40. I multiply this 40 by the other subdivision of the lira, that is by 12 *danari*, and I get 480. With a comma after the the 9 in the quotient I continue the division: 100 into 480 goes 4 times so I write the 4 in the quotient and have 80 to carry over. Thus we find that the decimal number 3,47 is almost equivalent to the complex number

Lire	***Soldi***	***Danari.***
3	9	4.

From which we see that we cannot always convert a decimal number into a perfectly equal complex number, but sometimes have to be content with an approximate number.

XVI. - The Rule of Three.

Q. What do we mean by the ***rule of three***?

A. We mean a way of solving problems by which, given three numbers, we look for a fourth which has the same relationship with one of them that the other two have between themselves.

For example, 3 workers do 12 metres of work in a day, and we want to know how many 7 would do in a day. As we can see we have three numbers, and we are looking for a fourth, that is, the number of metres that 7 workers can achieve. So we are looking for a number in relationship with the number 7 (workers), like the number 12 (metres) is in relation to the 3 (workers).

Q. How often do we need this rule?

A. In two cases:

1. When, given the value[15] of a determined number of units, we are looking for the value of another particular number.
So for example 3 workers do 12 metres of work in a day, so how many will 7 do? In this problem we have the value of a determined number of workers, that is we know that three workers are worth 12 metres, and we are looking for the value of another determined number of workers, that is, how many metres 7 workers are worth.
2. When, given a determined number of units and the value of each, we want to know how many units we can have with the same sum, but changing the value of the unit into another determined value.
So for example, with a certain sum I could buy 9 metres of material at lire 6 a metre, so how many could I buy with the same amount at L. 18 a metre?

Q. What rule do we use to resolve problems to do with the first case?

A. In the first case, knowing the value of a determined number of units, we have to first find the value of one unit by dividing the value of all by the number of units. The quotient will be the value of each. Then we multiply this quotient by the other number of units we are seeking the value for.

So in the given example we divide 12 by 3 and the quotient is 4 which indicates the work done by a worker. With this quotient 4, I multiply the other number of workers, that is the 7, since it is clear that 7 workers do 7 times 4 metres, and this way I get the fourth number sought, that is 28 metres of work that are done by 7 workers.

[15] Here 'value' is taken in its broad sense.

Q. How do we go about the second case?

A. In the second case, knowing a determined number of units and the value of one of these units, we begin by looking for the value of all the units, multiplying these two numbers by themselves, then we divide the product by the value of the other number of units we are looking for. The quotient will be the number we are looking for.

So in the given example, knowing the number of several units, 9 metres of bread, and the value of just one, which is lire 6, I begin looking for the value of all of them, which I get by multiplying these two numbers; 9 metres costs 9 times 6 lire, that is L. 54. When I have this product, I divide by lire 18 which is the value of each of the new units I am looking for, and it is clear that as often as 18 is contained in 54 is the number of metres there will be that I can buy. The quotient 3 will show that with L. 54, I can only get three metres if I have to pay L. 18 each.

EXERCISES

1. 1. If one load of bread m. 36 is worth 200 lire, how much will another of m. 40 cost?
2. 2. In a day 25 bricklayers did 57 m. c. of work; how many will 15 do?
3. 3. In 3 days 12 workers did some work, 36 workers would take how long then?
4. 4. There are 1500 soldiers in a fort with enough to eat for 6 months; how many soldiers would have to leave for this food to last two months more?

XVII. - Applications of the Rule of Three in problems concerning interest and simple Societies

Q. What do we mean by problems regarding ***interest***?

A. Interest problems are those regarding the income earned from an amount loaned to someone for each 100 lire, over a given time.

For example. John lends L. 1200 at 5 per 100, that is, with an agreement that he will be paid L. 5 for every 100 lire each year. He wants to know how much interest he gets annually.

Q. How many things do we have to consider in interest problems?

A. Four things need to be considered:

1. The amount received, called the ***capital;***
2. The ***tax*** or what the person owing has to pay for each 100 lire;
3. The ***time***, number of years, months and days for which the amount has been borrowed;
4. The ***earnings*** from the capital after a determined period.

N.B. The expression '5 per 100' is usually written as 5 %, so we would say 6 % to express 6 per cent, etc.

Q. How do we solve interest problems?

A. If we note it well, we find that it always comes back to one of the two cases of the *rule of three*; once we know which, we apply the rule.

So in the given example we see that it is the first case of the rule of three since we know the value or interest for a determined number of units, that is 100 lire brings in L. 5; and we are looking for the value or interest for another determined number of units, that is, for L. 1200. Therefore first we need to find out how much one lira earns. Dividing L. 5 percent, we find that the earnings from one lira is L. 0,05. We multiply 1200 with this quotient and we get an interest of L. 1200 at 5 percent which equals lire 60. If we then want to find the interest over more years, after getting the interest for one year we multiply this by the number of years. When we need to find the interest for months and days as well, we can begin by looking for the interest for one month or one day, the quotient, meaning the interest, is then multiplied by the number of months or days which we are looking for the interest for.

Q. What are problems of ***simple societies***?

A. These are problems regarding the way of dividing an amount which the society has earned or lost amongst its members.

Q. What do we have to consider in problems regarding simple societies?

A. Four things need to be considered:

1. Each member's ***contribution*** to the society;
2. The ***capital*** of the society which is the sum of the contributions;
3. The amount lost or earned, meaning ***total earnings or losses***;
4. ***How much the loss or gain affects each member***.

For example, two merchants set up a society; one contributes L. 3000, the other 5000. The earnings were L. 320. How much will each receive from this?

Q. How do we solve these problems?

A. These problems also can be converted into problems of the rule of three for the first case, the number of members, so follow the rules given for this. In the given example there are L. 320 which is the result of a determined number of units meaning the sum of the two contributions, so by adding these two together we begin by finding out how much one lira contributed to the society would earn; this is found by dividing the gains by the total contributed amount. We find L. 320 : 800 = 0,04 which is the result for each lira. With this number 0,04 we multiply each of the contributions separately and so we find L. 3000 × 0,04 = L. 120 per gain for the first contribution; for the 2nd contribution 5000 × 0,04 = 200 per gain.

EXERCISES.

1. What interest do we get from a capital of lire 5280 loaned at 6% for 15 years?
2. Find the interest for 10 thousand lire at 5% for 5 years 6 months.
3. What will be the interest on L. 6000 at 5% for 25 years?
4. A traveller spent 3 lire in food and sits down in the shade to eat his lunch: a friend joins him who had spent 5 lire and suggests they share. The other agrees; they are about to begin lunch when along comes a third, and he accepts and eats with them. When the third goes, he leaves them 8 lire for the food and their welcome telling the other two to divide it up. How much does each get?

5. Five people promise to help one another should one of them get into difficulties, each contributing according to their income. The first ears, each year. L. 1000; the second L. 1200; the third L. 1350; the fourth L. 1552; the fifth L. 2000. The first one loses L. 13540 in damage caused by fire. How much must the other four put in, supposing they wish to fully cover the damages? How much must each of the other 4 contribute supposing the same fire caused damages of L. 13640 to the second, third, fourth and fifth? How much does each have to contribute in each of these cases by giving only the agreed amount?

XVIII - DEFINITION of the Most Important Geometric Shapes

Q. What do we mean by ***Geometry***?

A. Geometry is a word made up of two Greek terms (*geo - metria*) which mean ***measuring the earth***; it is about properties and measures of extension.

Q. How many dimensions are there in finite extensions?

A. Three: ***length, breadth***, ***depth or height***, all essential attributes of bodies.

Q. What is a ***point***?

A. It is the end of a line without any dimensions.

Q. What is a ***line***?

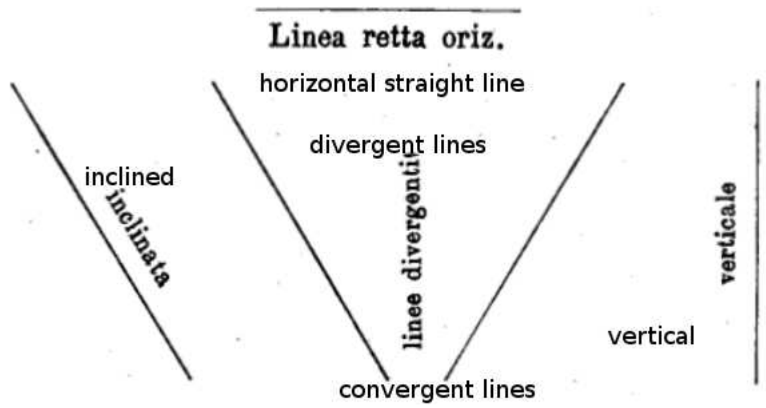
A. It is length, without breadth and depth.

Q. How is a line divided?

A. Into ***straight, curved and broken***.

Q. What is a ***straight*** line?

A. It is the shortest distance between two points. A straight line is horizontal if it follows the direction of a stagnant pool of water. It is called vertical if it follows the direction of thread with a lead weight. It is called inclined if it goes in any other direction. Two lines are said to converge if they meet. These same lines which converge at one point, diverge at another.



Q. What is a ***curved*** line?

A. Any line which is not straight, nor made up of straight liens, is called curved.

fig 1. 

Q. What is a ***broken*** line?

A. it is a line made up of straight lines.

Q. What is the ***surface***?

A. It is an extension in length and width without depth.

Q. What is the simplest of all surfaces?

A. It is the plane.

Q. What is a ***plane***?

A. It is a surface that can accept any kind of straight line.

Q. What is a curved surface? A. One that is not plane nor has plane surfaces.

Q. What is a solid or geometric body?

A. Everything that has the three dimensions of ***length, breadth and depth***.

Q. What is an angle.

A. It is the mutual inclination of two straight lines meeting at a point.

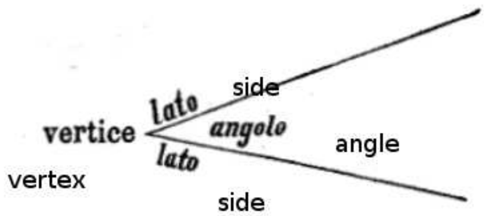


fig. 2.

Q. What do we call the point where two ***straight lines*** meet?

A. The ***vertex***.

Q. What do we call the two straight lines?

A. The two ***sides***.

Q. What is the ***perpendicular***?

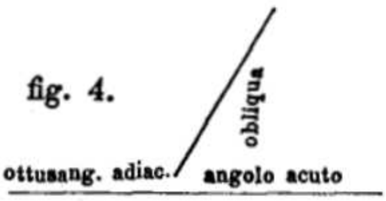
A. It is the straight line which meets another straight line such that the contiguous or adjacent angles are equal between them. they are called right angles.

fig. 3.

Q. What is an ***oblique*** line?

A. It is a straight line which when falling on another creates two ***adjacent angles*** which are not equal.

fig. 4.



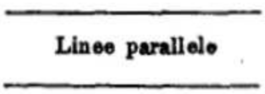
Q. What is the name of ***adjacent angles*** of an oblique line?

A. The one larger than a right angle is called ***obtuse***; the smaller one is called ***acute***.

Q. When can two straight lines be called parallel?

A. when they are both on the same plane and when prolonged indefinitely at both ends they can never meet.

fig. 5.



Q. What is a ***plane*** figure?

A. It is a plane closed all around by one or more lines.

fig. 6.

Q. What is the ***perimeter*** or boundary of the figure?

A. It is the sum of all the lines that enclose it.

Q. How are shapes classified?

A. Into ***rectilinear, curvilinear and free-form*** according to the lines that make it up: all straight, all curved, or part straight and part curved.

fig. 7.

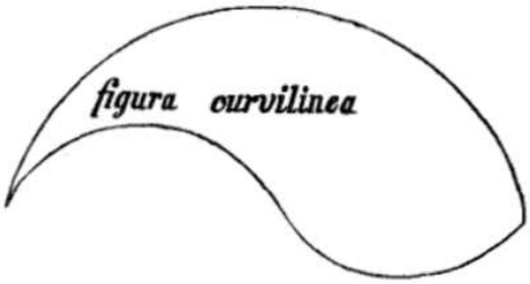


fig. 8.

Q. What are rectilinear shapes commonly called?

A. ***Polygons***.

Q. What do we call the lines making a perimeter?

A. Sides.

Q. What is the simplest of all polygons?

A. The ***triangle***.

Q. How are polygons classified?

A. As ***quadrilaterals*** or four-sided, ***pentagons*** with five, ***hexagons*** with six, ***hectagons*** with seven, ***octagons*** with eight, ***nonagons*** with nine, ***decagons*** with ten, ***dodecagons*** with twelve, ***pentadecagon*** with fifteen.

Q. How do we distinguish triangles?

A. By their sides and by their angles.

Q. How many are there by sides?

A. Three: ***equilateral*** with all sides equal, ***isosceles*** with two equal sides, ***scalene*** where all three are unequal.

Q. How many by angles?

A. Three: right angled triangle with one right angle: obtuse angled, with one obtuse angle: acute angled where all three are acute.

fig. 9.

fig. 10.

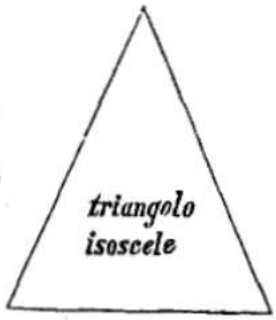


fig. 11.

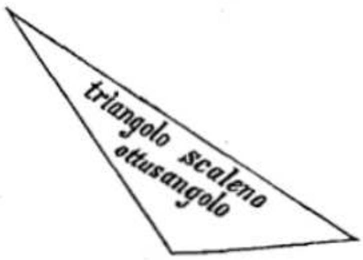


fig. 12.

Q. What do we mean by ***quadrilateral***?

A. A plain figure made up of four straight lines.

Q. How do we classify ***quadrilaterals***?

A. Into ***square, rectangular, rhombus, rhomboid and trapezium***.

Q. What is a square?

A. It is a quadrilateral with equal sides and all right angles.

fig. 13.

Q. What is a ***rectangle***?

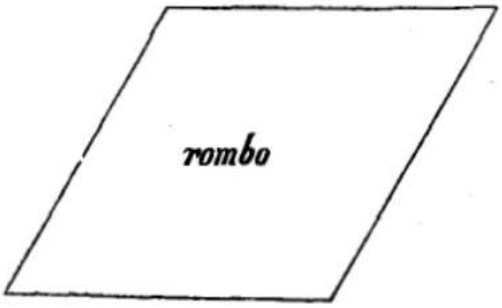
A. It is a quadrilateral with all right angles but without all equal sides.

fig. 14.

Q. What is a ***rhombus***?

A. It is a quadrilateral with all sides equal but without all right angles.

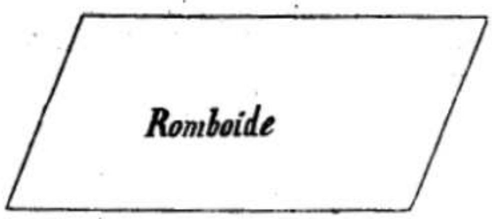
fig. 15.



Q. What is a ***rhomboid***?

A. It is a quadrilateral with only equal opposite sides without any right angle.

fig. 16.



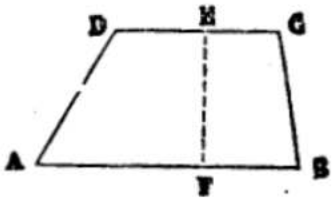
Q. What do we call these four quadrilaterals?

A. ***Parallelograms***, because opposite sides are parallel.

Q. What is a trapezium?

A. It is a quadrilateral but not a parallelogram.

fig. 17.

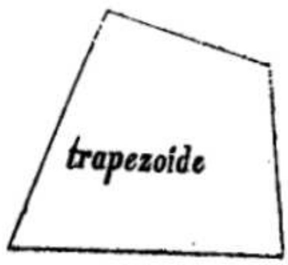


Trapezio.

Q. What are trapeziums commonly called?

A. A quadrilateral with only two parallel sides, (fig. 17). Trapeziums different from these are called trapezoids (fig. 18).

fig. 18.



Q. What is a ***diagonal***?

A. A diagonal is a line joining two non-consecutive vertices of a polygon.

fig. 19.

Q. What is a ***circle***?

A. It is a plane shape bounded by a curved line called a periphery or circumference which has all its points equidistant from an internal point called the centre.

fig. 20.

Q. What is the ***radius*** of a circle?

A. It is any line drawn from the centre to the periphery.

Q. What is the ***diameter*** of a circle?

A. it is any line that passes through the centre and terminates at opposite parts of the circumference.

Q. What is an ***arc***?

A. Any part of the circumference.

Q. What is a ***chord?***

A. The line that joins the two extremities of the arc.

Q. What is a s***egment***?

A. That part of the circle between the arc and the chord.

fig. 21.

Q. What is the ***sagitta***?

A. The line that divides the arc and the chord into two equal parts.

Q. What is a ***quadrant***?

A. An arc which equals a quarter of the circumference.

XIX. - Solid Geometry

Q. What is a ***solid or geometrical body***?

A. A solid or geometrical body is three-dimensional extension: ***length, breadth, height***.

Q. What is the ***volume*** of a body?

A. The volume of a body is the body itself, also the room it takes up or that we suppose it takes up.

Q. How many kinds of solids are there?

A. Two kinds, ***polyhedra*** and ***non-polyhedra***.

Q. What solids are called polyhedra or non-polyhedra?

A. ***Polyhedra*** are those whose surfaces are all plane, and ***non-polyhedra*** if they have a curved surface.

Q. Which are the main polyhedra?

A. The ***cube, prism*** and ***pyramid***.

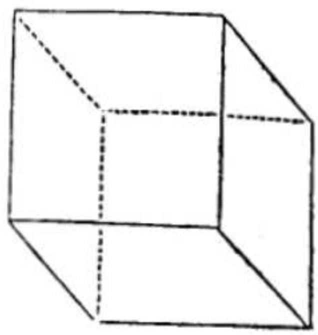
Q. Which are the main non-polyhedra?

A. the ***cylinder, cone*** and ***sphere***.

Q. What is a ***cube***?

A. The cube is a body bounded by six equal and square surfaces.

fig. 22.

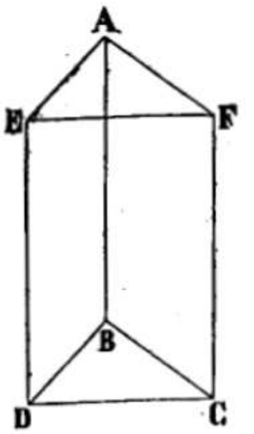


Cubo.

Q. What is a ***prism***?

A. a solid geometric figure whose two end faces are similar, equal, and parallel rectilinear figures, and whose sides are parallelograms.

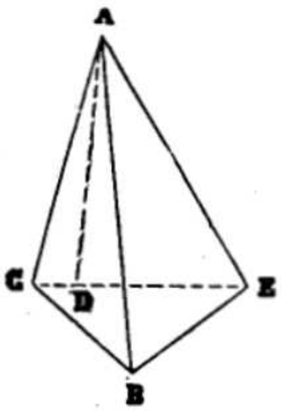
fig. 23.



Q. What is a ***pyramid***?

A. A solid object where: the base is a polygon (a straight-sided shape, the sides are triangles which meet at the top (the apex).

fig. 24.



Piramide.

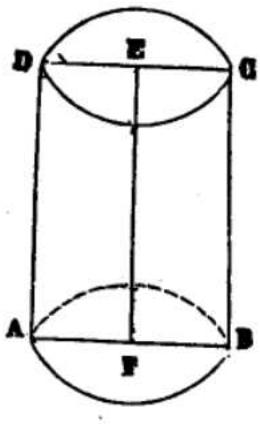
Q. What is a ***cylinder***?

A. A solid object with: two identical flat ends that are circular or elliptical and one curved side. fig 25. D G A B

Q. What is a ***cone***?

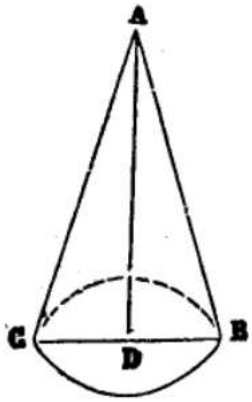
A. The cone is a solid or hollow object that tapers from a circular or roughly circular base to a point.

fig. 25.



Cilindro.

fig. 26.



Cono.

Q. What is a sphere?

A. The sphere is a round solid figure, or its surface, with every point on its surface equidistant from its center.

fig. 27.

Sfera.

Appendix I.

Table of Fixed Numbers and the Way to Use Them.

Q. What is the easiest way for us to get a clear idea of the new metric decimal system?

A. To get a clear idea of the new weights and measures we need to see which weights and measures can substitute the old ones, and which are equal to them, so it will be very helpful to read the following tables. They will help you keep an eye on the differences between Provinces. Tables of fixed numbers to convert old measures into new ones and vice versa, by simple multiplication.

Table 1 - Piedmont-Turin

DISTINZIONE DELLE MISURE	FATTORI O NUMERI FISSI per	RIDURRE LE MISURE ANTICHE IN NUOVE cioè	NUMERO DI CIFRE da separare NEL PRODOTTO	FATTORI O NUMERI FISSI per	RIDURRE LE MISURE NUOVE IN ANTICHE cioè	NUMERO DI CIFRE da separare NEL PRODOTTO
LINEARI	2·5	Le miglia in chilometri . .	una	4	I chilometri in miglia . . .	una
	3·09	trab. in metri.	due	·324	I metri in trab.	tre
	·514	I piedi in metri.	tre	1·944	I metri in piedi.	tre
	1·715	Le tese in metri.	tre	·583	I metri in tese.	tre
	·6	I rasi in metri.	una	1·67	I metri in rasi.	due
DI SUPERF.	9·526	I trab. quadr. in metri quad.	tre	·105	I metri quadr. in trab. quadrati.	tre
	·265	I piedi quadr. in metri quadr.	tre	3·779	I metri quadrati in piedi quadr.	tre
	·38	Le gior. in ett.	due	2·625	Le ettare in gior.	tre
	·381	Tavole in are.	tre	2·62	Are in tavole .	due
DI SOLIDITÀ	29·401	I trab. cubi in metri cubi .	tre	·034	I metri cubi in trab. cubi . .	tre
	·136	I piedi cubi in metri cubi .	tre	7·35	I metri cubi in piedi cubi . .	due
	5·041	Le tese pel fieno in steri . .	tre	1·198	Gli steri pel fieno in tese . . .	tre
	4·033	Le tese per legna in steri .	tre	·248	Gli steri per legna in steri .	tre
	4·08	Trab. camerale in metri cubi.	tre	0·245	I metri cubi in trab. camerali.	tre
DI CAPAC.	·23	Le emine in ettolitri . . .	due	4·34	Gli ettolitri in emine . . .	due
	·5	Le brente in ettolitri . . .	una	2·	Gli ettolitri in brente . . .	»
	2·3	Emina in decal.	una	·435	Decalitri in em.	tre
PESI	·9222	I rubbi in miriagrammi . .	quattro	1·0853	I miriagrammi in rubbi. . . .	quatt.
	9.223	Rubbi in kilog.	tre	·1085	Kilog. in rubbi.	quatt.
	·369	Le libbre in kilogrammi .	tre	2·711	I kilogrammi in libbre . . .	tre
	·306	Le oncie in ettogrammi .	tre	3·253	Gli ettogrammi in oncie . .	tre

Table 2: Lombard-Milan

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		METRICHE IN ANTICHE NUMERI FISSI		SUDDIVISIONI
di lung.	Bracc.	0,594936	m.	1,680852	M.	di 12 once, l'oncia di 12 punti.
	Piede	0,435185	»	2,297873	»	6 piedi fanno un trabucco.
di sup.	Pert. q.	6,545179	Are	0,152784	Ara	di 24 tav. che fan 3456 piedi q.
di cap.	Mogg.	1,462343	El.	0,683834	El.	di 8 staia, lo staio di 4 quartari.
	Soma	1,645136	»	0,607792	»	di 9 staia, lo staio di 4 quartari.
	Brenta	0,755544	»	1,323550	»	= 48 pinte = 96 boc. = 3 staia, lo staio di 2 em. l'emina di 2 q.
Pesi	Libbra grossa	0,762517	Cg.	1,311446	Cg.	di 28 once.
	Lib. pic.	0,326793	»	3,060040	»	di 12 once.
	Marco	0,234997	»	4,255370	»	di 8 once.
Monete	L. Austriaca	0,866	»			di 100 centesimi
	Fior.	2,48	»			di 2 metà, la metà di 2 quarti.

Table 3: Venice

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		SUDDIVISIONI
di lungh.	Braccio	0,6851 0,6384	m.	
	Piede	0,347398	»	di 12 once, l'oncia di 12 punti, il punto di 12 atomi; 5 piedi fanno il passo.
di sup.	Passo q.	0,030171	Are	di 25 piedi q.
	Campo	36,5661	»	
di cap.	Moggio	0,800	El.	di 4 staia, lo staio di 4 quarti, il quarto di 4 quartaroli.
	Secchio	0,1080	»	di 4 bozze, la bozza di 4 quartuzzi; 48 secchi fanno l'anfora.
di peso	Libbra gr.	0,476998	Cg.	di 12 once, l'oncia di 8 dramme.
	» sott.	0,302025	»	di 12 once, l'oncia di 144 carati.
Monete	le stesse della Lombardia.			

Table 4: Bologna

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		SUDDIVISIONI
di lungh.	Piede	0,380098	m.	di 12 once, l'oncia di 12 punti. La pertica era di 12 piedi.
	Braccio	0,640039	»	di 20 once.
di sup.	Tornatura	20,804358	Are	di 144 pertiche q. o tavole, la tavola di 100 piedi q.
di cap.	Corba (G.)	0,7864	El.	= 2 staia = 8 quartiroli = 32 quarticini.
	Corba	0,7859	»	di 4 quartarole, la quartarola di 15 boccali, il boccale di 4 fogliette.
di peso	Libbra	0,361850	Cg.	di 12 once, l'oncia di 8 ottavi l'ottavo di 20 carati, il carato di 4 grani.
Monete	Scudo	5,36	L.	di 10 paoli, il paolo di 10 baiocchi, il baiocco di 5 quattrini.

Table 5: Genoa

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		SUDDIVISIONI
di lungh.	Palmo	0,24808	m.	di 12 once, l'oncia di 12 punti, il punto di 12 atomi.
di sup.	Cannella	0,088625	Are	di 12 palmi superficiali, il palmo di 12 once superficiali.
di cap.	Emina (G).	1,2072	El.	di 4 staia, lo staio di 8 ottavi
	Barile da vino	0,7423	»	di 90 amole; 2 barili fanno la mezzarola.
	Barile da olio	0,6548	»	di 4 quarti.
di peso	Libbra gr.	0,348456	Cg.	
	» sott.	0,316678	»	di 12 once; 25 libbre fanno il *Rubbo*, 6 rubbi il *Cantaro*.
Monete	Lira antica	0,84	L.	

Table 6: Cagliari

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		SUDDIVISIONI
di lungh.	Trabucco	3,148200	m.	di 12 palmi, il palmo di 4 quarti.
	Raso	0,5993	»	
di sup.	Starello	39,867	Are	
di cap.	Starello (G).	0,48961	El.	di 19 imbuti.
	Pinta	0,8968	»	
di peso	Libbra	0,398985	Cg.	di 12 oncie.
Monete	Lira antica	1,88	L.	

Table 7: Parma

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		SUDDIVISIONI
di lungh.	Pertica	3,27100	m.	di sei braccia da muro, il braccio di 12 oncie, l'oncia di 12 punti.
	Braccio da seta	0,58775	»	
	» da tela	0,6395	»	
di sup.	Biolca	30,814390	Are	di 6 staia, lo staio dividesi in tav., piedi e oncie sempre di 12 in 12 (la tavola è di 4 pertiche q.).
di cap.	Staio (G).	0,4704	El.	di due mine, la mina di 8 quartarole.
	Crenta	0,71672	»	di 36 pinte, la pinta di due boccali.
di peso	Rubbo	8,2000	Cg.	di 25 libbre, la libbra di 12 oncie, l'oncia di 24 danari, il danaro di 24 grani.
Monete	Lira vecch.	0,20	L.	di 20 soldi.

Table8: Modena

MISURE	UNITÀ principali	ANTICHE in METRICHE NUMERI FISSI		SUDDIVISIONI
di lungh.	Piede	0,523048	m.	di 12 once; 6 piedi fanno il cavezzo o pertica.
	Braccio	0,633150	»	di 12 once.
di sup.	Biolca	28,364724	Are	di 72 tavole, la tav. di 4 cavezzi q.
di cap.	Staio (G.)	0,6325	El.	di due mine, la mina di 4 quarti.
	Quartaro	1,01812	»	di 90 boccali.
di peso	Peso	8,511425	Cg.	di 25 libbre, la lib. di 12 once, l'oncia di 12 ferlini.
Monete	Lira vecch.	0,305	L.	di 20 soldi, il soldo di 12 danari.

Table 9: Florence

MISURE	UNITÀ principali	ANTICHE in METRICHE NUMERI FISSI		SUDDIVISIONI
di lungh.	Braccio	0,583660	m.	di 12 soldi, il soldo di 12 danari; 5 braccia fanno la canna agrimensoria.
di sup.	Quadrato	34,061912	Are	si suddivide in tavole, pertiche, deche di 10 in 10; la deca vale 10 braccia q.
	Stioro	6,2341	»	di 12 panore, il pan. di 12 pugnora: il pugnoro vale braccia 9 87.
di solid.	Brac. cubo	0,19882	mc.	di 6 bracciuole, la bracc. di 12 once: 12 bracciole fanno il *traino* misura pel legname da costruzione, e 12 braccia cube fanno la *catasta* misura pel legname da ardere.
di cap.	Moggio (G).	5,847087	El.	di 8 sacca, il sacco di 3 staia, lo staio di 4 quarti, il quarto di 8 mezzette, la mezz. di 2 quartucci.
	Barile da vino	0,455840	»	di 20 fiaschi, il fiasco di 4 mezzette, la mezzetta di 2 quartucci.
	Barile da olio	0,334289	»	di 16 fiaschi, il fiasco di 4 mezzette, la mezzetta di 2 quartucci.
di peso	Libbra	0,339542	Cg.	di 12 once, l' oncia di 24 denari, il denaro di 24 grani.
Monete	Lira vecch.	0,840	L.	di 20 soldi, il soldo di 12 danari.
	Scudo	5,880	»	
	Fiorino	1,400	»	di 100 quattrini, il quattrino di 4 denari.

Table 10: Rome

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		SUDDIVISIONI
di lungh.	Palmo d'architetto	0,223422	m.	di 12 oncie l'oncia di 5 minuti, 10 palmi fanno la canna, lo staiuolo vale palmi 5,75, la catena 57,50
	Palmo mercantile	0,249	»	
	Piede	0,297896	»	$= \frac{4}{3}$ del palmo d' archit.: 5 piedi fanno il passo e 1000 passi il miglio.
di sup.	Pezza	26,4063	Are	= 6 catene q. = 4 quarte, la quarta è di 40 ordini, l'ordine di 10 staiuoli.
di cap.	Rubbio (G.)	2,944651	El.	= 4 quarti, il quarto di 4 scorzi, lo scorzo di 2 quartucci.
	Barile da vino	0,5834159	»	di 28 boccali, il boccale di 24 fogliette; la botte è di 16 barili.
	Barile da olio	0,5748059	»	
di peso	Libbra	0,339072	Cg.	di 12 once, l'oncia di 8 ottavi, l'ott. di tre denari, il denaro di 24 grani; 100 libbre fanno il quintale, 10 quintali il migliaio.
Mon^te	Scudo	5,36	L.	di 10 paoli, il paolo di 10 baiocchi, il baiocco di 5 quattrini

Table 11: Ancient Rome

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		SUDDIVISIONI
Linear.	Pes	0,296310	m.	= *palmi minores* = 1,333.... del *palmus maior* = 12 *unciae* = 0,2 del *passus maior* = 0,4 del *gressus* = 0,666... del *cubitus*. Lo *stadium* è di 625 piedi, l' *actus* di 120 e il *milliarum* di 5000.
di sup.	Iugerum	25,2839	Are	= 2 *actus* = $\frac{1}{2}$ dell' *heredium* = $\frac{1}{200}$ della *centuria* = $\frac{1}{800}$ del *saltus*. Il iugero ha l'area di 28,80 piedi q.
di cap.	Congius (G.)	0,3240	El.	= 6 *sextarii* = 12 *heminae* = $\frac{1}{8}$ dell' *amphora*.
	Amphora	0,25920	»	= 2 *urniae* = 3 *modii* = 6 *semodii* = $\frac{1}{20}$ *del culeus*.
di peso	Libra o as o pondo.	0,327182	Cg.	*di* 12 *unciae*; il *talentum* è di 80 libbre. Le altre misure di peso sono di 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 once, e di 3, 4, 5, 10, 100 assi; come indicano chiaramente i loro nomi stessi.
Monete	Asse librale	0,05	L.	= 2 *semisses* = 3 *trientes* = 4 *quadrantes* = $\frac{1}{10}$ del *denarius* = $\frac{1}{5}$ del *quinarius* = 0,4 del *semistertius*. Le ultime tre monete erano di argento e cominciarono a coniarsi nel 485 di Roma; l' asse invece e le sue suddivisioni erano di rame, e sono le monete più antiche dei Romani.
	Sesterzio	0,20	»	Unità d'uso dopo l' anno 563. Il *sestertiumorum*, cioè sestertiorum *millia*, valeva 1000 sesterzii, che fanno 200 lire italiane.

Table 13: Naples

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		SUDDIVISIONI
di lung.	Palmo	0,2651503	m.	= $\frac{1}{700}$ di un minuto primo del grado medio del merid. terrestre, si divide in dec. cent.; 10 palmi fanno una canna.
di sup.	Moggio	0,998684	Are	= 100 canne q., si divide in parti decimali.
di cap.	Tomolo (G).	0,5554511	El.	= 3 palmi cubi., si divide in 2 mezzette, la mezzetta in 2 quarti, il quarto in 6 misure
	Barile	0,4362503	»	di 60 caraffe, equivale a tre palmi cilindrici cioè ad un cilindro retto, largo un palmo e alto tre (16 barili fanno la botte).
di peso.	Rotolo	0,890997	Cg.	si divide in parti dec. I millesimi si chiamano trapesi (100 rotoli fanno un cantaio)
	Libbra	0,3207589	»	di 12 once.
Monete.	Ducato	4,25	L.	di 10 carlini, il carlino di 10 grana, il gr. di 12 cavalli.

Table 14: Palermo

MISURE	UNITÀ principali	ANTICHE IN METRICHE NUMERI FISSI		SUDDIVISIONI
di lungh.	Palmo	0,258098	m.	di 12 once, l'oncia di 12 linee, la linea di 12 punti. La canna è di 8 palmi.
di sup.	Salma	174,625873	Are	si divide in bisaccie, tomoli, mondelli, carozzi e quarti, sempre di 4 in 4; il quarto è di 4 canne q.
di cap.	Tomolo (G.)	0,1719305	El.	= 1 palmo cubo; si divide in mondelli, carozzi, quarti e quartigli di 4 in 4; (16 tomoli fanno la salma).
	Quartaro	0,1719305	»	= 1 palmo cubo; si divide in 20 quartucci, il quartuccio in 2 caraffe, la caraffa in due bicchieri, un barile vale 2 quartari, la botte 32.
di peso	Rotolo	0,79342	Cg.	di 30 once, l'oncia di 8 dramme, la dramma di 3 scrupoli, lo scrup. di 20 grani, il grano di 8 ottavi; (12 once fanno la libbra, 100 rotoli il cantaio)
Monete	Oncia	12,75	L.	di 30 tarì, il tarì di 20 grani, il grano di 6 piccoli. Un tarì siciliano vale un carlino di Napoli.

How to convert old measures into metric-decimal and back again using the preceding tables

Q. How can measures from the old system be converted into metric-decimal and vice versa?

A. Once you have found the fixed number, the conversion is done by means of multiplication.

Q. What do we mean by ***fixed number***?

A. By fixed number we mean the relationship between the weight or measure of one system with another other.

For example, if I want to find the fixed number or relationship between foot and metre, I will say: the foot is equal to 0,514 metres. This 514 (millimetres) is the fixed number or that part of the metre which corresponds to the length of the foot. If I want to find the relationship between metre and foot I say: the metre equals 1,944 feet, meaning a metre is one foot and nine hundred and forty four thousandth parts of a foot. The number 1,944 is the fixed number.

Q. What else must we watch out for with fixed numbers?

A. It is worth noting that since we have to convert wholes and fractions of the old system, to make the task easier we convert larger wholes into lesser ones. For example if we have *rubbi*, pounds and ounces, we convert *rubbi* into pounds, pounds into ounces, then do the conversion to weights in the new system.

Q. Given the fixed number, how can we convert measures from one system into the measures of the other?

A. Given the fixed number we convert the measures of one system into the other by multiplication, that is multiplying the fixed number by the number of goods that we want to convert, following the rules of decimal multiplication in each case.

EXAMPLE:

Four metres are how many feet?

Operation:

Fixed number or multiplicand	0,514
Number to convert or multiply	45
	2570
	2056
	23,130

Explanation:
514 is millimetres which are the length of a foot relative to a metre. There are 45 feet to be multiplied by the respective number 514. In the product we separate the three fraction digits. So we say: 45 feet is 23 metres plus 130 millimetres or 13 centimetres.

Q. How do we prove this operation?

A. The proof of this operation is done perfectly with the rule of swapping the factors around and multiplying them again.

Exercises on the Tables of Conversion of Measures

1. How many metres make 27 trabucchi?
How many *trabucchi*, feet and inches make 50 metres?
How many hours are equivalent to 7 days?

2.How many metres are five Milanese *bracci* (arms)?

How many *some* make 7 hectolitres?
...
(etc. for a further 13 exercises with measures from the various tables listed earlier)

Appendix II.

Comparison of currency from the various States of Europe and the Provinces of Italy with the new Lira or Franc.[16]

ITALY

Provincie Sarde.

Quadruplo di Genova	L.	70	00
Carlino	»	50	00
Doppia di Savoia	»	28	45
Doppietta	»	10	00
Scudo vecchio	»	7	10
Scudo di Sardegna	»	4	80
Lira	»	1	00
Reale	»	0	48
Soldo	»	0	05

Provincie di Lombardia.

Fiorino Austriaco	»	2	47
Lira Austriaca o svanzica nuova	»	0	86
Svanzica vecchia	»	0	83
Carantano	»	0	03

[16] The metric system in Europe was only introduced in France, Italy, Spain and Holland

Provincie di Parma.

Doppia	»	21 92
Ducato	»	5 15
Lira vecchia	»	0 20
Soldo	»	0 01

Provincie di Modena.

Scudo d'Ercole III	»	5 60
» di Francesco III	»	5 54
Ducato	»	2 80
Scudo dell'Aquila	»	1 42
Quarantana	»	0 65
Lira di Modena	»	0 305
Soldo o bolognino	»	0 015

Provincie di Toscana.

Francescone	»	5 60
Scudo Fiorentino	»	5 88
Pezza livornese	»	4 83
Franceschino	»	2 80
Fiorino	»	1 40
Lira vecchia	»	0 84
Lira lucchese	»	0 75
Soldo vecchio	»	0 042
» lucchese	»	0 37

Provincie della Romagna, Umbria e Marche.

Doppia	»	17 07
Scudo	»	5 32
Testone	»	1 59

89

Papetto	»	1 06
Paolo	»	0 532
Grosso o mezzo paolo	»	0 266
Baiocco	»	0 053

Provincie di Sicilia e Napoli.

Onza	»	12 75
Piastra	»	5 10
Ducato	»	4 25
Carlino Napoletano	»	0 425
Tarì siciliano	»	0 425

France

Scudo	»	5 80
Luigi	»	3 55
Lira antica detta tornese	»	0 99

England

Lira sterlina	»	25 208
Dollaro	»	5 41
Scellino	»	1 2604
Penny (Pence)	»	0 105
Farthings	»	0 026

Austria

AUSTRIA.

Tallero	»	5 19
Risdallero	»	4 96
Fiorino	»	2 59
Lira	»	0 867
Kreutzer	»	0 043

Prussia

Scudo o risdallero o tallero	»	3 71
Silbergroschen	»	1 20

Russia

Rublo	»	4 00
Solotnik	»	1 00
Dolo	»	0 047

Spain

Piastra	»	5 43
Reale di Plata	»	0 54
Realino	»	0 27

Portugal

Cruzada	»	2 95
Testone	»	0 61
Reis	»	0 006

ANCIENT CURRENCIES

Greece

Talento Attico d'oro	»	55608 98
» » d'argento	»	5560 89
» » d'Egína o di Corinto	»	9268 17
» » cominc. il 2° secolo avanti C.	»	5222 41

Rome

Aureus o solidus	»	20 38
Denarius	»	0 81
Quinarius	»	0 40
Sestertius o nummus	»	0 20
Dupondius	»	0 16
As, libella, o assipondius fino al 336 di R.	»	0 08
» » » 720 di R.	»	0 05
Sembella prima del 536 di Roma	»	0 04
Teruncius » » »	»	0 02
Sembella dopo il 536 di R.	»	0 025
Teruncius » » » »	»	0 015
Denaro sotto Augusto	»	0 79
» » Tiberio e Claudio	»	0 78
» » Nerone Galba e Domiz.	»	0 73

Il Talento di Babilonia valeva	»	7407 38
» » Mosè »	»	6172 82